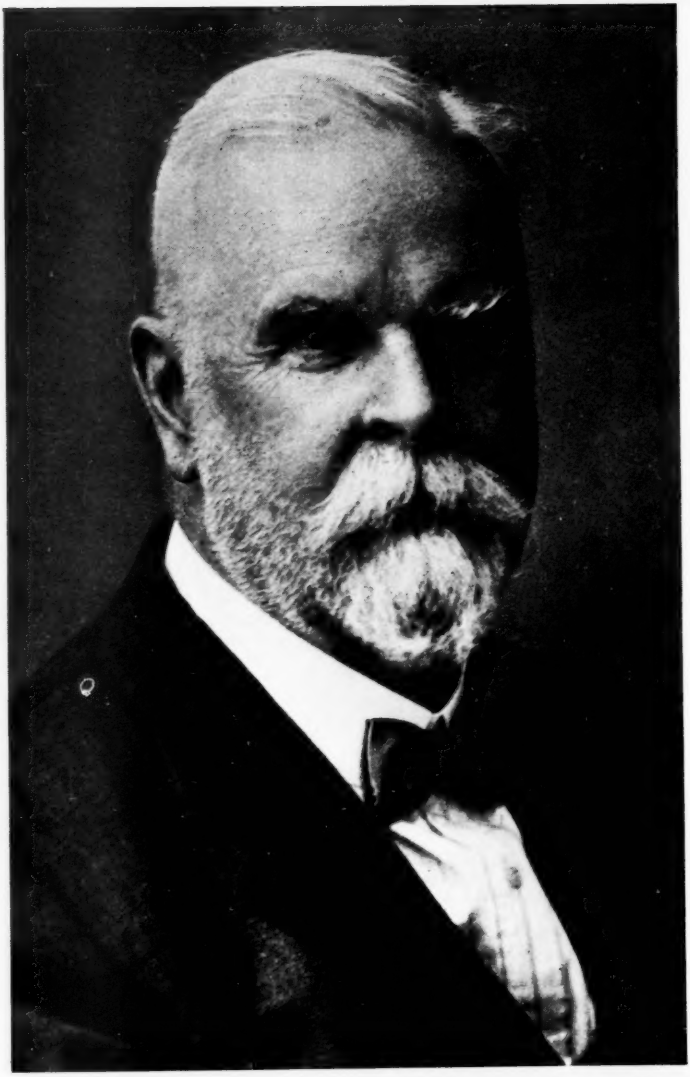

James Pierpont

JAMES PIERPONT—IN MEMORIAM

James Pierpoint died in San Mateo, California, on December 9, 1938, at the age of seventy-two. During the last year he had been in ill health.

James Pierpont belonged to an old New Haven family which has been associated with Yale in various ways. His forefather and namesake, the New Haven minister James Pierpont, was one of the founders of Yale College. James Pierpont's father, Cornelius Pierpont, was a wealthy business man in New Haven. He had four children, three sons and one daughter.

James, the second son, was born on June 16, 1866. He prepared for college at New Haven High School and then went to Worcester Polytechnic Institute. Originally he wanted to become a mechanical engineer, but it may be surmised that he gradually found that the practical applications of engineering appealed less to him than the study of the theoretical foundations of the subject. In any case, it must have been during his stay at Worcester that he discovered his love for the pure sciences because his plans were already laid. Shortly after his graduation in 1886 he left for Europe to begin a prolonged study of mathematics at various universities.

Most of the time he studied in Berlin and Vienna. In Berlin he became familiar with the works of Kronecker, and all of Pierpont's early papers show the profound influence of this algebraic school. The last years in Europe were spent in Vienna where he obtained his doctor's degree in 1894. Here he made many friends, and it seems that the "Gemütlichkeit" of university life in Vienna suited his own jovial temperament particularly well. His associations with Wirtinger and von Escherich were close and cordial and the friendships were renewed by Pierpont's occasional returns to Vienna. This period of studies was a very happy one, and in his later years Pierpont liked to reminisce about it, repeating often what a revelation the acquaintance with European mathematics and mathematicians had been to him.

After having received his Ph.D. degree Pierpont returned to New Haven and was appointed lecturer at Yale the same year, 1894. He was thoroughly imbued with the ideas of modern mathematics and he set to work with the zeal of a missionary to spread them among American mathematicians. In his first period he was particularly interested in algebra and equation theory. Together with Bôcher he gave the first Colloquium Lectures of the American Mathematical Society in Buffalo in 1896. Pierpont's topic was the Galois theory of equations. It was a set of lectures obviously inspired by Kronecker's and Weber's expositions, but Pierpont's presentation had, as most of his contributions, a clearness of its own which dissolved the complications of the theory. This colloquium lecture [7] is one of the few which have not been published in book form, and Pierpont sometimes expressed regret that this had not been done.* Pierpont was one of the outstanding leaders in the meetings of the Society in this period. After the meetings Bôcher, Osgood, and other friends often stopped in New Haven on their way home to continue the discussion of the problems which interested them.

Pierpont's academic career may be described in few words. At Yale he was rapidly promoted. In 1895 he was made an instructor, in 1896 assistant professor, and in 1898 professor in Yale College, a position which he held until his retirement in 1933. In 1923 he was appointed to the Erastus L. DeForest professorship in mathe-

* In the following the numbers refer to the list of Pierpont's printed articles appended to this article.

matics. It should also be mentioned that he was invited to lecture on algebraic numbers at Harvard 1899–1900 and during the summer session 1929 he lectured at the University of California at Berkeley. In 1909 he received an honorary LL.D. degree from Clark University.

Pierpont stimulated the advanced study of mathematics at Yale from the very first days. He attracted a number of graduate students, and the list of doctorates in mathematics increased soon after his appointment. The following represents a fairly complete list* of mathematicians who prepared for their Ph.D. degrees under his supervision: S. Kimura, W. A. Granville, G. T. Sellew, J. Westlund, W. M. Strong, L. M. Pierce, H. E. Hawkes, A. S. Gale, H. A. Merrill, C. E. Stromquist, E. L. Dodd, C. E. Smith, R. B. McClenon, J. C. Morehead, R. G. D. Richardson, G. E. Wahlin, F. J. Holder, E. B. Lytle, M. S. Walker, J. K. Lamond, E. W. Sheldon, H. L. Agard, I. Barney, B. H. Camp, J. L. Jones, W. A. Wilson, P. D. Schwartz.

Pierpont's teaching consisted mainly of graduate and more advanced undergraduate courses. His courses on Real and Complex Variables were long among the most important in the mathematics department. He enjoyed teaching and contact with the students, and he usually succeeded in transmitting some of his own enthusiasm to them. His lectures were forceful and clear and showed the impression of his wide knowledge of mathematics. At times this knowledge would throw him off on a tangent so that his lecture would end in fields far from the scheduled subject, to the delight of the enthusiastic students and to the grief of those whose interests were chained to the text of the day. He spent considerable time with the students, discussed and gave advice and helped them both mathematically and otherwise. He was one of the moving forces in the mathematics club and one of its most frequent and popular speakers. His activity at Yale can perhaps best be characterized by recalling G. D. Birkhoff's remark in his recent lecture on the history of American mathematics, where he considers "mathematicians who have shown the rare quality of leadership": "I would also mention with high esteem James Pierpont, who for many years was a source of inspiration at Yale." He was always interested in the improvement of the scientific standards of the department. He was mainly responsible for the appointment of E. W. Brown and also for the various other attempts made to attract leading mathematicians to Yale.

Pierpont was one of the most widely read men I have ever met. He could frequently be seen on the campus with his old-fashioned canvas book-bag over the shoulder, and during his close to forty years at Yale he carried a considerable part of the library in his bag. For many years he held the distinction of being the greatest borrower of books at the university library. His interest was in no way limited to mathematics. On the contrary his interests did not seem to have any limits at all. He devoured books on science, history, languages, geography, and travel with the same interest as novels and biographies.

Perhaps it might be worth while to recall a couple of his peculiar personal aptitudes. He seemed to have equal mastery over both hands. He could write with them both and it always used to impress the undergraduates when he drew illustrations on the blackboard using both arms simultaneously and independently. He could read and write upside down with great facility and I still remember vividly my surprise

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* The printed list of doctorates at Yale does not contain any indication of the name of the professor under whose supervision the thesis was written. The above list of names was obtained through the cooperation of some of the former students. Any corrections or additions would be welcome.

at the first conference with him across a table where he used this method in explaining his formulas.

Pierpont enjoyed friends, company, and conversation. Walking was his only exercise. He loved a good discussion and when one came to see him it usually would not be long before one was drawn into some argument. Even if one at times would disagree with him, it was impossible not to admire his wide knowledge and memory. Personally he was always very modest about his own achievements. At times he could show a burst of temper, mostly in cases where matters of principle were involved, sometimes in meetings of the department or faculty. He usually regretted it afterwards but this did not prevent him from showing the same temper in the same questions shortly afterwards. This was quite contrary to his informal dealings with the students, where he did not like to enforce the rules too strictly; "the individual before the institution" was his usual comment in such cases.

In the recent Semicentennial Publications one finds ample evidence of Pierpont's devotion to the American Mathematical Society and his constant interest in its progress. He was elected a member in 1894 after his return from Europe. It should be mentioned that he served as a member of the Council for the periods 1899–1901, 1927–1929, 1931–1933, and was elected vice-president in 1905. He also was a nominee of the American Mathematical Society to the National Research Council and a member of the committee on the semicentennial celebration. One should mention particularly Pierpont's work in connection with the foundation of the *Transactions of the American Mathematical Society* in the period 1898–1900. He spent considerable effort and time in travel and conferences to pave the way for the new publication. Beside his more material contributions he also added a considerable amount of optimism which was needed at this time since there were grave doubts whether there would be an adequate supply of memoirs to keep such a publication in existence. The successful financing of the *Transactions* during the first period was in no small degree due to Pierpont's efforts.

Pierpont presented a large number of papers to the Society at its various meetings. During his first years as a member his contributions to the *Bulletin* were particularly numerous. During his later years one will perhaps remember him best from the several invited addresses which he gave with his usual clarity in his own eloquent and entertaining manner. His colloquium lectures at the Buffalo meeting have already been mentioned. At the summer meeting at Wellesley in 1921 he gave the address: *Some mathematical aspects of the theory of relativity.* In 1925 he was invited to give the Gibbs lecture at Kansas City and spoke on: *Some modern views of space.* At the annual meeting in Nashville in 1927 he gave a very interesting lecture on: *Mathematical rigor, past and present.* This lecture was later translated and published in the Revista Matemática Hispano-Americana. At the annual meeting in New York in 1928 he discussed: *On the motion of a rigid body in a space of constant curvature,* and at Berkeley in 1929 he gave the address: *Non-Euclidean geometry, a retrospect.*

Let us finally turn to Pierpont's contributions to mathematics. He was always very modest, but he had the true scientist's pleasure in his work. A new result delighted him and he had to communicate it and discuss it with his associates, and it usually would not be long before he presented it in a lecture to the mathematics club at Yale.

While his production touches upon many domains it may suitably be divided into three separate periods. During the first period 1894–1900 Pierpont was largely interested in algebra and the theory of equations, directly inspired by his early

German training. Most of his publications in this time are expository, concerned with the development of the Galois theory of equations, construction of regular polygons and modular equations [1, 4, 5, 6, 7, 10]. His first paper, published in the Monatshefte für Mathematik und Physik, contains a quite exhaustive history of the quintic equation. His own contributions to the theory are in most cases only new methods of presentation and modifications of proofs [2, 9, 12]. In this connection one should mention, however, two later more important papers connected with the theory of equations [30, 39]. The first gives a generalization of the secular equation and provides a simple proof for the fact that all its roots are real. This paper has been reproduced in Bieberbach's *Algebra*. The second paper is concerned with an equation occurring in electron theory and provides an opportunity for a critical discussion of the various approximation methods for the determination of the complex roots of an algebraic equation.

A number of popular expositions written around the turn of the century still make interesting reading, for instance: *Mathematical instruction in France* [14], and *On the arithmetization of mathematics* [11], later translated into Polish. At the Congress of Arts and Sciences at the Universal Exposition in St. Louis in 1904 Pierpont gave an address on *The history of mathematics in the nineteenth century*.

The second period (1900–1920) in Pierpont's mathematical production is characterized by his interest in the theory of functions of a real variable and integration. To this period belong the papers [17, 19, 20,] and his two textbooks: *Lectures on the Theory of Functions of a Real Variable*, dedicated to Andrew Phillips, and *Functions of a Complex Variable*, dedicated to von Escherich. It is difficult to form an opinion of the importance of Pierpont's contribution to the theory of integration, but it seems established that he independently created a theory of integration which in many ways was similar to Lebesgue's theory and in some ways different from it, but he had the misfortune of seeing it anticipated by the classical investigations of Lebesgue. Certain objections to Pierpont's theory published by Fréchet seem to be due mainly to a misconception of the meaning of Pierpont's terminology. A remark by Pierpont in his reply may be of historical interest: "To be historically accurate, I had no intention whatever of generalizing Lebesgue's integrals. When years ago I hit on my definition of integration, I did not know how it was related to Lebesgue's theory. I found out later that when the field of integration is measurable my integrals are identical with Lebesgue's and I have therefore called them Lebesgue integrals throughout my book."

The last period in Pierpont's production falls after 1920 and is entirely inspired by the theory of relativity. In a large number of papers he considers the properties of non-euclidean geometries. The resulting mechanical problems are discussed in detail, for instance various gravitational problems, Foucault's pendulum, the motion of a rigid body about a fixed point. The theory of optics in spaces of constant curvature attracted him particularly, and he even went into the specialized problems of lenses and refractors in such spaces. He prepared a book on non-euclidean geometry, but the manuscript does not seem to have been completed.

The published works of James Pierpont

1. *Zur Geschichte der Gleichung des* V. *Grades* (*bis* 1858), Monatshefte für Mathematik und Physik, vol. 6 (1895), pp. 15–68.

2. *On the invariance of the factors of composition of a substitution group*, American Journal of Mathematics, vol. 18 (1896), pp. 153–155.

3. *Note on C. S. Peirce's paper on "A quincuncial projection of the sphere,"* American Journal of Mathematics, vol. 18 (1896), pp. 145–152.

4. *Lagrange's place in the theory of substitutions*, this Bulletin, vol. 1 (1894–1895), pp. 196–204.

5. *On an undemonstrated theorem of the "Disquisitiones Arithmeticae,"* this Bulletin, vol. 2 (1895–1896), pp. 77–83.

6. *On the Ruffini-Abelian theorem*, this Bulletin, vol. 2 (1895–1896), pp. 200–221.

7. *Galois' theory of algebraic equations* (reproduction with slight alterations of parts of a course of lectures at the Buffalo Colloquium of this Society in September, 1896), I. *Rational resolvents*, Annals of Mathematics, (2), vol. 1 (1899–1900), pp. 113–143; II. *Irrational resolvents*, ibid., vol. 2 (1900–1901), pp. 22–56.

8. *Maxima and minima of functions of several variables*, this Bulletin, vol. 4 (1897–1898), pp. 535–539.

9. *On modular equations*, this Bulletin, vol. 3 (1896–1897), pp. 279–292.

10. *Early history of Galois' theory of equations*, this Bulletin, vol. 4 (1897–1898), pp. 332–340.

11. *On the arithmetization of mathematics*, this Bulletin, vol. 5 (1898–1899), pp. 394–406.

11a. *On the arithmetization of mathematics* (translation of [11] into Polish by S. Dickstein), Wiadomosci Matematyczne, vol. 3, pp. 249–263.

12. *On elliptic functions*, this Bulletin, vol. 5 (1898–1899), pp. 490–492.

13. *The summer meeting of the Deutsche Mathematiker-Vereinigung at Munich, September*, 1899, this Bulletin, vol. 6 (1899–1900), pp. 282–287.

14. *Mathematical instruction in France*, this Bulletin, vol. 6 (1899–1900), pp. 225–249.

15. *Galois theory of equations*, Encyclopedia Americana.

16. *The history of mathematics in the nineteenth century* (address delivered before the department of mathematics of the International Congress of Arts and Science, St. Louis, September 20, 1904), this Bulletin, vol. 11 (1904–1905), pp. 136–159.

17. *On multiple integrals*, Transactions of this Society, vol. 6 (1905), pp. 416–434.

18. *Lectures on the Theory of Functions of Real Variables*, vol. 1, Ginn, 1905, xii+560 pp.

19. *On improper multiple integrals*, Transactions of this Society, vol. 7 (1906), pp. 155–174.

20. *Area of curved surfaces*, Transactions of this Society, vol. 7 (1906), pp. 489–498.

21. *The Theory of Functions of Real Variables*, vol. 2, Ginn, 1912, xii+645 pp.

22. *Functions of a Complex Variable*, Ginn, 1914, xiv+583 pp.

23. *Reply to Professor Fréchet's article*, this Bulletin, vol. 22 (1915–1916), pp. 289–302.

24. *A reply to a reply*, this Bulletin, vol. 23 (1916–1917), pp. 174–175.

25. *Geometric aspects of Einstein's theory*, Annals of Mathematics, (2), vol. 23 (1921–1922), pp. 228–254.

26. *The geometry of Riemann and Einstein*, Part I, American Mathematical Monthly, vol. 30 (1923), pp. 425–438; Part II, ibid., vol. 31 (1924), pp. 26–39.

27. *Non-euclidean geometry from non-projective standpoint*, Proceedings of the International Congress of Mathematicians, Toronto, 1924, vol. 1, pp. 117–128.

28. *Some modern views of space* (the Gibbs Lecture for 1925), this Bulletin, vol. 32 (1926), pp. 225–258.

29. *Note on horospheres*, this Bulletin, vol. 32 (1926), pp. 525–528.

30. *On a generalization of the secular equation*, this Bulletin, vol. 33 (1927), pp. 294–296.

31. *On an application of Bouguer's theorem*, American Mathematical Monthly, vol. 34 (1927), pp. 134–135.

32. *Optics in hyperbolic space*, Transactions of this Society, vol. 30 (1928), pp. 33–48.

33. *Optics in space of constant non-vanishing curvature*, American Journal of Mathematics, vol. 49 (1927), pp. 343–354.

34. *Classification of quadrics in hyperbolic space*, American Journal of Mathematics, vol. 49 (1927), pp. 143–151.

35. *On the geometry whose absolute is a ruled quadric*, Monatshefte für Mathematik und Physik, vol. 35 (1928), pp. 111–128.

36. *Mathematical rigor, past and present*, this Bulletin, vol. 34 (1928), pp. 23–53.

37. *El rigor matemático pasado y presente* (translation of [36] into Spanish), Revista Mathemática Hispano-Americana, Part I, vol. 3, pp. 181–192; Part II, vol. 3, pp. 208–216.

38. *Note on Einstein's equation of an orbit*, this Bulletin, vol. 34 (1928), pp. 582–584.

39. *On the complex roots of a transcendental equation occurring in the electron theory*, Annals of Mathematics, (2), vol. 30 (1929), pp. 81–91.

40. *Foucault's pendulum in elliptic space*, Transactions of this Society, vol. 31 (1929), pp. 444–447.

41. *On the motion of a rigid body about a fixed point in space of constant curvature*, American Journal of Mathematics, vol. 51 (1929), pp. 287–294.

42. *A note on Foucault's pendulum*, American Mathematical Monthly, vol. 36 (1929), pp. 161–162.

43. *On the attraction of spheres in elliptic space*, this Bulletin, vol. 35 (1929), pp. 351–356.

44. *Non-euclidean geometry, a retrospect*, this Bulletin, vol. 36 (1930), pp. 66–76.

45. *Cayley's definition of non-euclidean geometry*, American Journal of Mathematics, vol. 53 (1931), pp. 117–126.

OYSTEIN ORE

THE APRIL MEETING IN DURHAM

The three hundred fifty-eighth meeting of the Society was held at Duke University, Durham, North Carolina, on Friday and Saturday, April 7–8, 1939. The attendance included the following one hundred forty-seven members of the Society:

C. R. Adams, R. B. Adams, Leon Alaoglu, Warren Ambrose, W. L. Ayres, C. L. Bacon, Reinhold Baer, R. H. Bardell, G. D. Birkhoff, Richard Brauer, E. T. Browne, Herbert Busemann, A. V. Bushkovitch, W. D. Cairns, E. A. Cameron, A. D. Campbell, C. E. Carey, Leonard Carlitz, C. L. Carroll, C. R. Cassity, J. W. Cell, C. E. Clark, J. M. Clarkson, A. B. Coble, L. W. Cohen, Nancy Cole, M. J. Cox, H. B. Curry, D. C. Dearborn, B. F. Dostal, Arnold Dresden, F. G. Dressel, L. A. Dye, J. J. Eachus, W. W. Elliott, Paul Erdös, F. A. Ficken, K. W. Folley, Orrin Frink, A. S. Galbraith, C. F. F. Garis, L. L. Garner, T. S. George, J. J. Gergen, P. W. Gilbert, A. O. Gray, J. A. Greenwood, T. N. E. Greville, B. L. Hagen, D. W. Hall, Marshall Hall, J. E. Hart, Philip Hartman, A. O. Hickson, R. C. Hildner, Einar Hille, M. P. Hollcroft, T. R. Hollcroft, V. A. Hoyle, Witold Hurewicz, Nathan Jacobson, R. F. Johnson, B. W. Jones, H. A. Jordan, Mark Kac, E. R. van Kampen, J. L. Kelley, J. R. F. Kent, W. W. Landis, A. E. Landry, J. W. Lasley, Solomon Lefschetz, Jack Levine, J. B. Linker, W. R. Longley, E. L. Mackie, H. M. MacNeille, Dorothy Manning, A. V. Martin, W. T. Martin, E. J. McShane, E. R. C. Miles, Max Morris, W. L. Morris, R. C. Morrow, D. S. Morse, H. T. Muhly, C. G. Mumford, D. C. Murdoch, H. M. Nahikian, A. L. Nelson, I. M. Niven, Rufus Oldenburger, L. F. Ollmann, J. M. H. Olmsted, F. W. Owens, H. B. Owens, H. V. Park, A. M. Pegram, B. J. Pettis, C. S. Pettis, C. G. Phipps, M. H. Plass, W. T. Puckett, E. S. Quade, W. C. Randels, J. F. Randolph, B. P. Reinsch, R. G. D. Richardson, J. H. Roberts, G. de B. Robinson, Samuel Saslaw, C. H. W. Sedgewick, R. W. Sedgewick, D. M. Seward, J. A. Shohat, R. G. Simond, Abraham Sinkov, R. E. Smith, V. E. Spencer, O. H. Stecker, R. W. Stokes, J. D. Tamarkin, J. H. Taylor, M. E. Taylor, J. M. Thomas, E. W. Titt, C. B. Tompkins, C. C. Torrance, M. M. Torrey, W. J. Trjitzinsky, A. W. Tucker, J. W. Tukey, Oswald Veblen, L. I. Wade, R. J. Walker, A. D. Wallace, Henry Wallman, W. G. Warnock, C. W. Watkeys, Albert Wertheimer, Hermann Weyl, G. T. Whyburn, A. L. Whiteman, Norbert Wiener, Oscar Zariski, H. S. Zuckerman.

This meeting was held in connection with the celebration of the centennial of the origins of Duke University. At the invitation of a special committee acting jointly for the Society and Duke University, the following three addresses were given on Friday morning, Friday afternoon, and Saturday morning, respectively: *Configurations defined by theta functions*, Professor A. B. Coble; *Invariants*, Professor Hermann Weyl; *The ergodic theorem*, Professor Norbert Wiener. The presiding officers at these three sessions were Professor J. M. Thomas, Dean G. D. Birkhoff, and Vice President C. R. Adams, respectively. General sessions for short papers with Professors Orrin Frink and Arnold Dresden presiding followed the addresses on Friday morning

and afternoon. On Saturday morning after the invited address there was a brief business session followed by sections for papers in Topology and in Analysis with Professors W. L. Ayres and H. M. MacNeille presiding.

All sessions were held in the Chemistry Building and in the Physics Building.

On Friday afternoon tea was served to the mathematicians and their guests in the Ball Room of the Men's Union. At the dinner in the Men's Union on Friday evening, there were two speakers, President W. P. Few of Duke University and Professor Oswald Veblen. The dinner was attended by one hundred seventy-nine members and guests.

The local committee for this meeting arranged many interesting features which included a carillon recital, a showing of the film *Grand Illusion*, trips to a tobacco factory and a hosiery mill, and a luncheon at Carolina Inn in Chapel Hill. The Society adopted a resolution, presented by Professor A. D. Campbell, expressing its appreciation and thanks to the President of the University and the members of the Mathematics Department for their competent arrangements and cordial hospitality.

The Council met on Friday, April 7, at 8:30 P.M. in the Administration Building.

The Secretary announced the election of the following five persons to membership in the Society:

Mr. Frank Aloysius Becker, St. Mary's University, San Antonio, Texas;
Miss Beatrice Miller Edison, New York University;
Mr. Samuel J. Loring, Chance Vought Aircraft Company, East Hartford, Conn.;
Mr. Manuel Salazar y Arce, National Railways of Mexico, Estacion Colonia, FF.CC. Nacional, Mexico, DF;
Mr. Winston M. Scott, University of Alabama.

It was reported that the following two persons had been elected as nominees on the Institutional Memberships of the following institutions:

Brown University: Professor Otto Neugebauer.
Western and Southern Life Insurance Company: Dr. Jerome G. Lemmer (West Baden College of Loyola University, West Baden Springs, Ind.).

The following appointment by President Evans was reported: as a Committee on the Award of the Cole Prize in Algebra, Professors E. T. Bell (chairman), Emile Artin, and Saunders MacLane.

The 1939 midwestern Thanksgiving Meeting is to be held on December 1–2 at the University of Missouri; the Annual Meeting of 1940 at Louisiana State University.

Titles and cross references to the abstracts of papers read at this meeting follow below. The papers were read in the various sections as follows: papers numbered 1 to 8, General Session, Friday morning; papers 9 to 14, General Session, Friday afternoon; papers 15 to 23, session for Topology, Saturday morning; papers 24 to 31, session for Analysis, Saturday morning; and papers 32 to 56, whose abstract numbers are followed by the letter *t*, were read by title. Of those presenting papers Mr. Rodabaugh was introduced by Professor W. P. Ott, Professor Barone by Professor L. A. Dye, and Mr. Schwartz by Professor J. A. Shohat. Paper 9 was read by Professor Martin, paper 12 by Dr. Tompkins, paper 13 by Dr. Galbraith, paper 16 by Dr. Youngs, and paper 30 by Dr. Spencer.

1. H. T. Muhly: *Valuations and infinitely near algebraic loci.* Preliminary report. (Abstract 45-5-216.)

2. Marshall Hall: *The position of the radical in an algebra.* (Abstract 45-5-181.)

3. Rufus Oldenburger: *Completely reducible forms.* (Abstract 45-5-218.)

4. Nathan Jacobson: *The fundamental theorem of the Galois theory for quasi-fields.* (Abstract 45-3-139.)

5. Richard Brauer: *On groups whose order is divisible by the first power of a prime.* Preliminary report. (Abstract 45-5-164.)

6. Reinhold Baer: *Duality and commutativity of groups.* (Abstract 45-5-155.)

7. L. A. Dye: *A Cremona transformation associated with the rational normal curve of order n in* S_n. (Abstract 45-5-174.)

8. T. N. E. Greville: *Invariance of the admissibility of variates under certain general types of transformations.* (Abstract 45-5-180.)

9. R. H. Cameron and W. T. Martin: *Reciprocals of Fourier-Stieltjes transforms analytic in a strip.* (Abstract 45-5-165.)

10. J. A. Shohat: *On the Lagrange interpolation formula based on the zeros of orthogonal Tchebycheff polynomials.* (Abstract 45-5-239.)

11. E. J. McShane: *On multipliers for Lagrange problems.* (Abstract 45-5-208.)

12. Marston Morse and C. B. Tompkins: *The existence of minimal surfaces of general critical type.* II. (Abstract 45-5-215.)

13. A. S. Galbraith and S. E. Warschawski: *On the convergence of Sturm-Liouville series.* (Abstract 45-5-177.)

14. Einar Hille: *Contributions to the theory of Hermitian series.* (Abstract 45-5-188.)

15. J. L. Kelley: *A note on the arc-wise connectivity theorem.* (Abstract 45-5-196.)

16. Tibor Radó and J. W. T. Youngs: *On upper semicontinuous collections.* (Abstract 45-5-225.)

17. W. T. Puckett: *The images of 2-dimensional surfaces under 0-regular transformations.* (Abstract 45-5-222.)

18. G. T. Whyburn: *On the existence of certain transformations.* (Abstract 45-5-254.)

19. J. H. Roberts: *A note on topological mappings.* (Abstract 45-5-231.)

20. A. D. Wallace: *Some cyclicly extensible and reducible properties.* (Abstract 45-5-249.)

21. C. R. Cassity: *Two theorems on the rational quartic surface in space of four dimensions.* Preliminary report. (Abstract 45-7-263.)

22. L. D. Rodabaugh: *On the partial differential equation* $\partial z/\partial x + f(x, y)\partial z/\partial y = 0$. (Abstract 45-7-276.)

23. D. W. Hall (National Research Fellow): *On new characterizations of the 2-sphere.* Preliminary report. (Abstract 45-7-268.)

24. H. G. Barone: *Limit points of sequences and their transforms by methods of summability.* (Abstract 45-5-157.)

25. F. G. Dressel: *A class of solutions for the heat equation.* Preliminary report. (Abstract 45-5-172.)

26. Philip Hartman: *On Dirichlet series involving random coefficients.* (Abstract 45-5-183.)

27. I. M. Niven: *On a certain partition function.* (Abstract 45-5-217.)

28. L. W. Cohen: *On the mean ergodic theorem.* (Abstract 45-5-169.)

29. W. C. Randels: *On the absolute summability of Fourier series.* II. (Abstract 45-5-229.)

30. J. A. Shohat and Vivian E. Spencer: *Mechanical quadratures coefficients as functions of the moments.* (Abstract 45-5-240.)

31. D. M. Seward: *Note on an integral equation.* (Abstract 45-5-238.)

32. E. E. Betz: *On accessibility and separation by simple closed curves.* (Abstract 45-7-259-*t*.)

33. Alonzo Church: *On the concept of a random sequence.* (Abstract 45-3-134-*t*.)

34. J. M. Feld: *Polygons as fundamental elements in the geometry of plane cubic curves.* (Abstract 45-5-176-*t*.)

35. P. G. Hoel: *A significance test for minimum rank in factor analysis.* (Abstract 45-5-189-*t*.)

36. E. V. Huntington: *Stirling's formula with remainder.* (Abstract 45-5-190-*t*.)

37. Edward Kasner and J. J. DeCicco: *A characterization of the Moebius group of circular transformations.* (Abstract 45-5-195-*t*.)

38. Norman Levinson: *An inequality in function theory.* (Abstract 45-3-141-*t*.)

39. A. N. Lowan: *On some problems in the diffraction of heat.* (Abstract 45-5-203-*t*.)

40. Saunders MacLane: *The universality of formal power series fields.* (Abstract 45-5-204-*t*.)

41. P. T. Maker: *The ergodic theorem for a sequence of functions.* (Abstract 45-5-205-*t*.)

42. E. J. McShane: *The condition of Legendre for double integral problems of the calculus of variations.* (Abstract 45-5-209-*t*.)

43. J. F. Ritt: *On ideals of differential polynomials.* (Abstract 45-3-144-*t*.)

44. M. S. Robertson: *The variation of the sign of v for an analytic function u+iv.* (Abstract 45-5-232-*t*.)

45. O. F. G. Schilling: *Units in p-adic algebras.* (Abstract 45-5-234-*t*.)

46. H. M. Schwartz: *A study of a certain class of continued fractions.* (Abstract 45-5-235-*t*.)

47. W. E. Sewell: *Integral approximation and continuity in the complex domain.* (Abstract 45-3-147-*t*.)

48. W. E. Sewell: *Integral approximation and continuity in the real domain.* (Abstract 45-3-146-*t*.)

49. C. W. Vickery: *Deformations of frequency functions.* (Abstract 45-5-245-*t*.)

50. C. W. Vickery: *Random and biased sampling machines.* (Abstract 45-5-246-*t*.)

51. A. D. Wallace: *On ergodic continua.* (Abstract 45-5-248-*t*.)

52. A. D. Wallace: *An axiomatic treatment of separation.* Preliminary report. (Abstract 45-7-279-*t*.)

53. J. L. Walsh: *On interpolation by functions analytic and bounded in a given region.* (Abstract 45-5-250-*t*.)

54. Morgan Ward: *Finite point lattices.* (Abstract 45-1-102-*t*.)

55. Morgan Ward: *Note on the general rational solution of the equation* $ax^2-by^2=cz^3$. (Abstract 45-1-103-*t*.)

56. Morgan Ward: *The algebra of lattice functions.* (Abstract 45-1-105-*t*.)

T. R. HOLLCROFT,
Associate Secretary

THE APRIL MEETING IN CHICAGO

The three hundred fifty-ninth meeting of the American Mathematical Society was held at the University of Chicago on Friday and Saturday, April 14–15, 1939. This was the largest spring meeting in Chicago in the history of the Society, the attendance being about two hundred fifty including the following one hundred seventy-seven members of the Society:

A. A. Albert, J. V. Atanasoff, W. L. Ayres, I. A. Barnett, Walter Bartky, E. F. Beckenbach, J. H. Bell, S. F. Bibb, K. E. Bisshopp, G. A. Bliss, Morris Bloom, Henry Blumberg, L. M. Blumenthal, M. G. Boyce, Richard Brauer, J. L. Brenner, F. L. Brooks, O. E. Brown, P. B. Burcham, R. S. Burington, L. E. Bush, W. H. Bussey, H. H. Campaigne, E. W. Chittenden, R. V. Churchill, M. D. Clement, Max Coral, J. J. Corliss, A. R. Crathorne, E. L. Crow, D. R. Curtiss, H. T. Davis, J. E. Davis, W. M. Davis, Douglas Derry, L. E. Dickson, W. J. Dixon, J. M. Dobbie, J. L. Doob, R. H. Downing, D. M. Dribin, R. J. Duffin, J. J. Eachus, M. C. Eide, Churchill Eisenhart, C. G. Erickson, H. P. Evans, C. J. Everett, H. S. Everett, L. R. Ford, Bernard Friedman, Morris Friedman, D. G. Fulton, H. L. Garabedian, B. E. Gatewood, H. A. Giddings, H. H. Goldstine, G. D. Gore, Cornelius Gouwens, L. M. Graves, Louis Green, F. L. Griffin, L. W. Griffiths, P. R. Halmos, W. L. Hart, Olaf Helmer, M. R. Hestenes, T. H. Hildebrandt, J. J. L. Hinrichsen, D. L. Holl, A. S. Householder, D. H. Hyers, M. H. Ingraham, Dunham Jackson, S. B. Jackson, E. D. Jenkins, R. N. Johanson, Chosaburo Kato, R. B. Kershner, P. W. Ketchum, J. M. Kinney, S. C. Kleene, C. F. Kossack, W. C. Krathwohl, A. C. Ladner, E. P. Lane, R. E. Langer, C. G. Latimer, R. A. Leibler, D. C. Lewis, W. C. McDaniel, C. C. MacDuffee, Ralph Mansfield, H. W. March, Morris Marden, R. G. Mason, A. E. May, J. R. Mayor, A. N. Milgram, W. E. Milne, U. G. Mitchell, W. L. G. Mitchell, M. G. Moore, C. W. Moran, E. J. Moulton, F. H. Murray, Tadasi Nakayama, Albert Neuhaus, C. V. Newsom, E. A. Nordhaus, M. J. Norris, M. A. Norval, Rufus Oldenburger, L. F. Ollmann, R. S. Park, G. A. Parkinson, J. F. Paydon, E. R. Peck, G. H. Peebles, Sam Perlis, R. S. Phillips, G. B. Price, Tibor Radó, G. Y. Rainich, E. D. Rainville, R. B. Rasmusen, W. T. Reid, C. E. Rickart, F. D. Rigby, R. F. Rinehart, D. H. Rock, W. H. Roever, W. E. Roth, R. G. Sanger, Max Sasuly, A. C. Schaeffer, I. J. Schoenberg, G. E. Schweigert, W. T. Scott, C. E. Sealander, C. G. Shover, H. A. Simmons, C. B. Smith, F. C. Smith, W. N. Smith, E. S. Sokolnikoff, I. S. Sokolnikoff, R. H. Stark, R. C. Stephens, B. M. Stewart, Otto Szász, H. P. Thielman, R. M. Thrall, L. W. Tordella, Leonard Tornheim, H. C. Trimble, A. R. Turquette, E. H. Umberger, E. P. Vance, V. J. Varino, J. I. Vass, H. E. Vaughan, R. W. Wagner, H. S. Wall, J. A. Ward, L. E. Ward, K. W. Wegner, M. E. Wescott, F. J. Weyl, G. W. Whaples, L. R. Wilcox, F. B. Wiley, K. P. Williams, L. A. Wolf, M. C. Wolf, Y. K. Wong, F. E. Wood.

On Friday morning there were two sections, Algebra, Professors I. A. Barnett and C. G. Latimer presiding; and Analysis, Professor I. S. Sokolnikoff presiding. On Friday afternoon Professor M. H. Ingraham gave the Symposium Lecture on *Rational methods in matrix equations*. Professor L. E. Dickson presided at this lecture. On Satur-

day morning there were again two sections, Analysis with Professor Dunham Jackson presiding, and Geometry and Topology with Professor E. W. Chittenden presiding.

On Friday afternoon after the Symposium Lecture the ladies of the Department of Mathematics of the University of Chicago served tea in the Commons Room of Eckhart Hall.

There was a dinner for members and guests on Friday evening with an attendance of one hundred thirty-eight. Professor C. C. MacDuffee of the University of Wisconsin acted as toastmaster and called on Professor Richard Brauer of the University of Toronto and Professor F. L. Griffin of Reed College, who brought greetings to the group from the Canadian and Pacific Coast mathematicians. He then called on Professor L. E. Dickson who is retiring from the staff at the University of Chicago at the end of the year. Professor Dickson spoke of the pressing need of readable mathematical books in English.

Titles and cross references to the abstracts of the papers read at this meeting follow below. Papers numbered 1 to 9 were read before the Algebra section, papers 10 to 20 before the section for Analysis Friday morning, papers 21 to 31 before the Analysis section Saturday morning, and papers 32 to 42 before the section for Geometry and Topology. Papers 43 to 57 whose abstract numbers are followed by *t*, were read by title. Mr. Yuan Lay was introduced by Professor G. Y. Rainich, Dr. A. E. Heins by Professor William Marshall, Mr. Fulton Koehler by Professor Dunham Jackson, Mr. M. H. Heins by Professor J. L. Walsh. Paper 6 was read by Professor Rainich, 7 by Dr. Duffin, 15 by Dr. Beckenbach, 21 by Professor Barnett, 22 by Dr. Scott, 39 by Mr. Reichelderfer.

1. J. A. Ward: *Theory of analytic functions in linear associative algebras.* Preliminary report. (Abstract 45-5-252.)

2. C. G. Latimer: *The complete solution of certain Diophantine equations.* (Abstract 45-5-200.)

3. D. M. Dribin: *Class field theory of solvable algebraic number fields.* (Abstract 45-5-173.)

4. H. C. Trimble: *On the ring of matrices commutative with a given matrix.* (Abstract 45-3-148.)

5. R. M. Thrall: *Trilinear forms and determinental manifolds.* Preliminary report. (Abstract 45-5-243.)

6. Yuan Lay: *On the imbedding of the skew part into an associative algebra.* (Abstract 45-5-201.)

7. R. J. Duffin and A. C. Schaeffer: *A problem in metric diophantine approximation.* (Abstract 45-3-136.)

8. Douglas Derry: *On finite abelian p-groups.* I. (Abstract 45-3-135.)

9. H. H. Campaigne: *Product hypergroups.* (Abstract 45-5-167.)

10. A. E. Heins: *The solution of the discrete equation of heat conduction.* (Abstract 45-5-184.)

11. H. W. March: *Infinite plane strip of orthotropic material under a concentrated load.* (Abstract 45-5-206.)

12. W. C. McDaniel: *The deflection of an orthotropic plate.* Preliminary report. (Abstract 45-5-207.)

13. Walter Bartky: *Solution of systems of differential equations by definite integrals.* (Abstract 45-5-158.)

14. R. V. Churchill: *On an extension of the Sturm-Liouville development.* (Abstract 45-5-168.)

15. E. F. Beckenbach and Maxwell Reade: *A characterization of plane isothermic maps.* (Abstract 45-5-159.)

16. E. D. Rainville: *Linear operational equations and the Laplace transformation.* (Abstract 45-5-226.)

17. Fulton Koehler: *Orthogonal polynomials on certain algebraic curves.* (Abstract 45-5-198.)

18. G. H. Peebles: *The boundedness of certain systems of orthogonal functions.* (Abstract 45-5-219.)

19. Dunham Jackson: *Orthogonal polynomials on curves of the second degree.* (Abstract 45-5-192.)

20. J. L. Doob: *The law of large numbers for continuous stochastic processes.* (Abstract 45-5-171.)

21. I. A. Barnett and Otto Szász: *On a certain Diophantine equation.* (Abstract 45-5-156.)

22. W. T. Scott and H. S. Wall: *Power series in which each exponent is at least twice the preceding.* (Abstract 45-5-237.)

23. R. S. Phillips: *Integration in a convex linear topological space.* (Abstract 45-5-220.)

24. Olaf Helmer: *A theorem of the Picard type.* (Abstract 45-5-186.)

25. L. M. Graves: *The Weierstrass condition for multiple integral problems of the calculus of variations.* (Abstract 45-3-138.)

26. M. R. Hestenes: *On the first necessary condition for minima of double integrals.* (Abstract 45-7-269.)

27. M. G. Moore: *On fundamental systems of solutions for linear difference equations.* (Abstract 45-5-213.)

28. W. T. Reid: *A note on the Du Bois-Reymond equations in the calculus of variations.* (Abstract 45-5-230.)

29. H. H. Goldstine: *Minimum problems in the functional calculus.* (Abstract 45-5-178.)

30. Otto Szász: *On the order of the partial sums of Fourier power series.* (Abstract 45-5-242.)

31. A. C. Schaeffer: *The Fourier-Stieltjes coefficients of a function of bounded variation.* (Abstract 45-3-145.)

32. F. J. Weyl: *Exponential curves.* (Abstract 45-5-253.)

33. R. B. Kershner: *Ergodic curves.* (Abstract 45-5-197.)

34. L. F. Ollmann: *On joining finite subsets of a planar Peano space by simple closed curves.* (Abstract 45-3-143.)

35. H. E. Vaughan: *Locally peripherally compact spaces.* (Abstract 45-5-244.)

36. Tibor Radó: *On cyclic elements.* (Abstract 45-5-223.)

37. G. E. Schweigert: *A note on the limit of orbits.* (Abstract 45-5-236.)

38. E. P. Vance: *Generalizations of non-alternating and non-separating transformations.* (Abstract 45-3-149.)

39. Tibor Radó and P. V. Reichelderfer: *Some properties of continuous transformations in the plane.* (Abstract 45-5-224.)

40. L. R. Wilcox: *A theorem on curves in a projective space.* (Abstract 45-5-255.)

41. G. D. Gore: *Transformations for two classes of surfaces.* (Abstract 45-5-179.)

42. A. N. Milgram: *Partially ordered sets: bounds and mappings.* (Abstract 45-7-272.)

43. R. E. O'Connor: *Quadratic and linear congruence.* (Abstract 45-3-142-*t*.)

44. Morgan Ward and R. P. Dilworth: *Evaluations over residuated structures.* (Abstract 45-1-106-*t*.)

45. Morgan Ward: *A characterization of Dedekind structures.* (Abstract 45-1-100-*t*.)

46. C. C. Camp: *A multiple series expansion.* Preliminary report. (Abstract 45-5-166-*t*.)

47. A. A. Aucoin and W. V. Parker: *Diophantine equations whose members are homogeneous.* (Abstract 45-5-154-*t*.)

48. E. D. Rainville: *Linear partial differential operators and the Laplace transformation.* (Abstract 45-5-227-*t*.)

49. E. D. Rainville: *Relations among certain operators of class three.* (Abstract 45-5-228-*t*.)

50. R. S. Phillips: *On additive set functions.* (Abstract 45-5-221-*t*.)

51. M. H. Ingraham: *An algorithm for the solution of the unilateral matrix equation.* (Abstract 45-5-191-*t*.)

52. Marie M. Johnson: *An extension of a covariant differentiation process.* (Abstract 45-5-193-*t*.)

53. V. W. Adkisson: *Plane peanian continua with homeomorphisms extendible in the sense of Antoine.* (Abstract 45-5-152-*t*.)

54. M. H. Heins: *Extremal problems for functions analytic and single-valued in a doubly connected region.* (Abstract 45-5-185-*t*.)

55. L. M. Blumenthal: *Metric methods in determinant theory.* (Abstract 45-5-163-*t*.)

56. M. R. Hestenes and H. H. Goldstine: *The condition of Mayer for the problem of Bolza in the calculus of variations.* (Abstract 45-5-187-*t*.)

57. C. F. Kossack: *The existence of collectives in abstract space.* (Abstract 45-5-199-*t*.)

W. L. AYRES,
Associate Secretary

THE APRIL MEETING AT STANFORD UNIVERSITY

The three hundred sixtieth meeting of the American Mathematical Society was held at Stanford University on Saturday, April 15, 1939. A general session was held Saturday morning and another in the afternoon. Professors H. F. Blichfeldt, A. D. Michal, Gabor Szegö, and W. M. Whyburn presided at various times. At the morning session Professor T. Y. Thomas of the University of California at Los Angeles gave an address entitled *Embedding theorems in differential geometry*. The attendance included the following thirty-two members of the Society:

H. M. Bacon, Clifford Bell, B. A. Bernstein, H. F. Blichfeldt, C. R. Bubb, F. W. Dresch, C. A. Hayes, E. R. Hedrick, R. D. James, Hans Lewy, A. T. Lonseth, Rhoda Manning, W. A. Manning, A. E. Marston, A. D. Michal, C. B. Morrey, W. H. Myers, C. D. Olds, A. C. Olshen, R. W. Rector, R. M. Robinson, E. L. Ross, W. H. Simons, Pauline Sperry, Gabor Szegö, T. Y. Thomas, J. V. Uspensky, R. K. Wakerling, W. F. Whitmore, W. M. Whyburn, B. C. Wong, Max Zorn.

On Saturday afternoon a tea for members of the Society and their friends was given by Professor and Mrs. Szegö at their home.

Titles of papers read at the meeting follow. Those whose abstract numbers are followed by the letter *t* were read by title.

1. R. K. Wakerling: *On the loci of* $(k+1)$*-secant k-spaces of a curve in r-space.* (Abstract 45-5-247.)

2. R. A. Favila: *A surface with doubly stratifiable directrices.* (Abstract 45-5-175.)

3. B. A. Bernstein: *Groups and abelian groups in terms of negative addition.* (Abstract 45-5-161.)

4. Max Wyman: *Non-holonomic covariant vector fields.* (Abstract 45-5-256.)

5. C. B. Morrey: *Multiple Denjoy-Perron integrals.* (Abstract 45-5-214.)

6. A. T. Lonseth: *The problem of Plateau in hyperbolic space.* (Abstract 45-5-202.)

7. Max Zorn: *Menger-Brouwer dimension in subanalytic spaces.* Preliminary report. (Abstract 45-5-258.)

8. A. D. Michal and A. B. Mewborn: *General projective differential geometry of paths.* (Abstract 45-5-210.)

9. L. F. Walton: *On the theory of ideal numbers.* Preliminary report. (Abstract 45-5-251.)

10. H. A. Arnold: *Note on completely continuous differentials.* (Abstract 45-5-153-*t*.)

11. R. M. Robinson: *On numerical bounds in Schottky's theorem.* (Abstract 45-5-233-*t*.)

12. O. G. Harrold: *Invariance of the dimensionality of a compact metric space under certain continuous transformations.* (Abstract 45-5-182-*t*.)

13. Max Wyman: *Postulates for the determination of a non-holonomic linear connection.* (Abstract 45-5-257-*t*.)

14. P. M. Swingle: *A finitely containing connected set.* (Abstract 45-5-241-*t*.)

15. Harlan C. Miller: *A separation theorem.* (Abstract 45-5-211-*t*.)

16. Harlan C. Miller: *On the characterization of a certain type of continuous curve.* (Abstract 45-5-212-*t*.)

17. Clifford Bell: *Solution of numerical equations.* (Abstract 45-5-160-*t*.)

18. A. D. Michal and A. B. Mewborn: *General flat projective geometry.* (Abstract 45-7-271-*t*.)

T. M. PUTNAM,
Associate Secretary

ROOM ON DETERMINANTAL LOCI

The Geometry of Determinantal Loci. By T. G. Room. Cambridge, University Press; New York, Macmillan, 1938. 28+483 pp.

The original purpose of this volume, as stated in the preface, was to provide a systematic account, according to the method and scope of Reye's projective geometry, of the properties of the loci in higher space which are analogous to quadric and cubic surfaces of ordinary space. But these limitations were too narrow; in the final form, synthetic and algebraic methods are used freely, frequently combined in the same problem.

While part of the work is original with the author, and general account of recent contributions, especially of the English School, has been taken, the general scope and procedure of the book are those developed from three sources:

Baker, *Principles of Geometry*, 6 volumes;

Bertini, *Geometria Proiettiva degli Iperspazi*;

Segre, *Mehrdimensionale Räume* (Encyklopädie, III, C7).

Usually citations are not given to original sources, but to the treatment of the problem under consideration in one or another of the three treatises mentioned. Some important recent contributions are not cited at all.

The subject is introduced by concrete examples. Every algebraic plane curve can be defined by a determinant equated to zero. The eliminant of the parameters in two projective pencils of planes defines a quadric surface. The two systems of generators appear at once from its form. Similarly, the equation of a general cubic surface can be so written. Lines on the surface, properties of the double-six, and the plane representation are immediate consequences. Similarly, the space cubic curve is expressed by a two by three matrix of rank one; its systems of bisecants and of tangents are now apparent.

The notation and elaborate symbolism are explained at length. It is a curious fact that almost every locus discussed in the literature on algebraic geometry can be expressed by determinants. Many properties of one locus are shown to be projections or sections of another in a space of a different number of dimensions. Joins, meets, duals (in various spaces) have their usual meanings, and are employed freely, with an appropriate symbolism.

The three fundamental characteristics of any algebraic locus are its dimension, order, and its freedom. The symbol $(|p, q|_r, [n])$ is used to denote the locus of meets of the sets of corresponding linear spaces $[n-p+r]$ of q related stars $]p-1[$ in $[n]$. The symbol $|p, q|$ means a matrix of p rows, q columns, each element being a linear form in point coordinates in linear space of n dimensions. The number r is the rank of the matrix.

Each such locus can be generated in two ways by systems of projective primes (loci defined by one linear equation in point coordinates), and may have multiple loci of fewer dimensions. This establishes a (1, 1) correspondence among the points of one locus, or between the points of two different loci. Thus is opened a new vista of Cremona transformations. However, with few exceptions these are not further developed in the book. It does, on the other hand, use this scheme freely and with skill in developing the representation of a locus upon a linear space.

The plan of the book is to develop a few general theorems and then to elaborate the properties for particular values of p, q, r, n, and to show consequences of restricted

positions of the bases of the generating systems. The effort is made to include all the forms already cited in the literature, and to show relations among results that were derived independently of each other.

The book is hardly suited for systematic reading, but rather as a handbook to assist in marshalling all the pertinent facts concerning a particular problem. In this sense it will frequently be of great value. Among the properties so developed are the pairing theorems and the theorem of the double-N. The double-six of lines on a cubic surface furnishes a simple but typical example.

The general manifolds contain relations among the elements of the p, q matrix, and there may be relations among the coefficients of the defining primes. If $n=pq-1$, the elements of the matrix can be taken as the faces of the simplex of reference, and there are no such relations. Such manifolds are called key manifolds. The dual configurations now appear as polar reciprocals as to a quadric primal referred to a self-polar simplex. In this case the pairing theorems and the double-N take on a more symmetric form.

A chapter on determinantal primals, $p=q$, $r=p-1$ shows that when $p>3$, the primal is particular. Thus, for $n=3$, a quartic surface requires one condition to be so written. It contains a general sextic curve of genus three. When $p=n$, a determinantal primal can be mapped upon $[p-1]$.

The last chapter of Part I is concerned with the number of degrees of freedom of projectively generated manifolds. The results are correct for the case of independent elements, but are not always trustworthy for restricted systems. The author carefully points out the limitations of the procedure.

The second part, about 230 pages, is concerned with particular cases; matrices of special forms, symmetric determinants, and so on. The first chapter deals with Veroneseans of quadrics, that is, loci defined by referring the totality of quadrics in $[q]$ to the primes of $q(q+1)/2$. Particular cases arise when the given system has base elements of various orders and dimensions. The next chapter is concerned with symmetric, partly symmetric, and polar manifolds, Lüroth primals, defined as the sum of reciprocals of linear forms in point coordinates, and the associated Cremona inversion, together with a detailed discussion of Coble's group of reciprocal inversions defined by $n+3$ points in $[n]$. The chapter ends with the treatment of manifolds defining normal elliptic curves of order n. Skew-symmetric determinants and Grassmannians of lines are next taken up, including manifolds generated by the duals of lines, which the author calls secunda.

A chapter on normal rational scrolls of $[k]$'s deals with generalizations of ruled surfaces. Thus, for $k=0$, the curve results. It can be defined by determinants of order 2. The dual is the normal rational developable. For $k=1$, ruled surfaces of lines result; these are given several pages, followed by more general scrolls. A chordal $[s-1]$ is a linear space meeting a scroll in s points. This includes the system of bisecants of a rational normal curve, and more generally defines involutions of order s on the curve.

The $[k]$'s of $[m]$ can be represented by the points of $[(m-k)(k+1)]$, the base elements being normal rational scrolls. This chapter is a generalization of the representation of lines of $[3]$ by points of $[4]$. A long chapter on association and incidence theorems is a direct generalization of Segre's famous fifth associated line theorem, which may be formulated as follows: if a, b, c, d are four arbitrary lines of $[4]$, and a', b', c', d' are the transversal lines of sets of three of them, then the four $[3]$'s a, a', and so on have a common line e. The five lines a, b, c, d, e form a symmetric set; any plane which meets four of them also meets the fifth. In the same manner the Segre cubic primal in $[4]$ is generalized and various prime representations of it discussed.

This chapter closes with a summary of the theory of three associated normal rational curves in [4], and its generalizations.

Part III, about 100 pages, is concerned with determinantal primals in [4], and loci contained on it. The first is the Bordiga sextic surface, locus of the meets of corresponding primes of four related sheaves. It contains ten lines, and there are ten planes meeting it in cubic curves which with the lines form a double-ten. The surface is mapped upon a plane. Upon the surface is a system of trisecant lines, one line of which passes through any point of [4]. This at once furnishes a map of the system of trisecants on [3].

Then follow special cases, including those surfaces having a triple point, and others having restricted base points in the representation. A similar surface of order 10 in [8] is generated by [6]'s in four related sheaves. The projection from any four points on it into [4] is the Bordiga sextic. The next chapter considers primals in [4] defined as the vanishing of a general determinant of order 4, the elements of which are arbitrary linear forms in point coordinates. This primal has twenty nodes, and contains four simply infinite systems of lines h_i, which are discussed in detail. A Bordiga sextic surface on the primal also lies on a cubic primal; the residual intersection is another Bordiga surface. The two systems are distinct and triply infinite.

The primal is mapped upon a [3] by means of its generating systems and the Cremona transformation defined by four bilinear equations among the point coordinates of two [4]'s is discussed. Primes in either space are transformed into general determinantal quartic primals of the other.

The last chapter considers various special cases, including those having a triple point, those containing planes, Veneroni's primal, those defined by a symmetric determinant, polar primals.

Three short appendices are devoted respectively to Veroneseans of primals, the characteristics of various manifolds, and the freedom of manifolds. The volume is provided with an index.

The style is strikingly simple. There are numerous repetitions in description, but these are necessary to prevent ambiguity. At first the reader gets the impression that the symbolism is cumbersome and too much insisted upon, but when he has mastered it, it is at once apparent that descriptions would be much more complicated without it.

The printing and presswork are excellent and the proofreading practically faultless. Two incorrect subscripts are the only cases that might cause confusion.

May this volume, which contains the essence of the entire literature on the subject, be of service compatible with the care and devotion spent in its preparation.

VIRGIL SNYDER

SHORTER NOTICES

Some Notes on Least Squares. By W. E. Deming. Washington, D.C., The Graduate School, Department of Agriculture, 1938. 181 pp.

The author is conservative in calling this work *Some Notes on Least Squares.* It may not be what one would call a finished product judged by the standards of textbook requirements, but a reading will show it to be much more than the skeleton outline of a series of lectures on least squares.

At the outset we find what he calls *fragments in the history of least squares* which presents a fair picture of its development from Gauss' time, and with the copious references throughout the text one gets a thumbnail bibliography of the subject brought up to the present.

The treatment of the subject is logical and consistent throughout. It may be likened to what students of harmony call "variations on a theme"; and the author's theme is *The Minimizing of the Sum of the Squared Residuals* as enunciated by Gauss in his "theoris motus" (1809). It is interesting to note that in all the problems in least squares considered he begins the attack with this principle, adapting it to the special case by simple mathematical stages. In fact, he almost apologizes for the necessary use of the first derivative!

Believing that, in the present day, least squares cannot be disassociated from statistical researches that have been made since Gauss, the author has woven some of it into the material comprising these notes.

It is regrettable that this method of treatment does not suggest to the reader that, after all, the whole scheme is based on probability; that no mention is made of the probable error (as a modification of mean square error) still used by a considerable number of research and technical men in connection with the normal curve.

After a few simple applications in finding the best value from several direct measures, the author takes up the General Problem in Least Squares for which he gives a complete solution, whether it is a problem in curve fitting with adjustable parameters or one involving geometrical conditions or a combination of both cases, with all measured quantities subject to error.

The solution is made possible, of course, by approximation, since approximate values of the unknowns are necessary and the application of the principle of Gauss leads up to the normal equations involving corrections to the approximations as unknowns. There is nothing new in this idea, but most problems result in equations which are either solved with great difficulty or else defy analysis so that a clarifying device must be used if definite results are hoped for.

Commenting on this phase the author states—"All problems in least squares theoretically can be solved without the use of Lagrange multipliers. Occasionally it may be easier to dispense with them, but the truth is that most problems become hopelessly involved. The elegance and simplicity that they lend to all problems seem to me sufficient to displace all other possible methods in the design of a routine procedure. If Kummell in 1879 had introduced Lagrange multipliers, he would surely have accomplished the general solution that he was looking for."

This solution may be easily adjusted to any set of conditions simple or complex: for instance, if some of the measured quantities are free from error, this is taken care of by the simple expedient of making their respective weights infinite.

In the section on curve fitting the plan is outlined for obtaining the best values

for the parameters, their weights and errors, the adjustment of the observations, and the confidence belts associated with the curve.

Lastly there are interesting exercises and notes on the formation of the normal equations for various functions to best fit certain measurements such as straight line, parabola, exponential, exponential with a lineal component, the generalized hyperbola together with three examples in curve fitting completely worked out, viz.:

Example 1, fitting an isotherm $y=a+bx+cx^2+dx^4$ with parameters subject to the condition $x=1$ when $y=1$.

Example 2, the polynomial $y=a+bx+cx^2$ with both x and y subject to error.

Example 3, an example useful in forestry, fitting $x=ay^bz^c$ where $x=$volume of a tree, $y=$merchantable height of tree, and $z=$diameter at breast height, the data having been secured from 66 trees.

This treatise is to be commended for its completeness and for maintaining the single purpose of all work in least squares, the minimizing of $\sum \overline{res}^2$. Although the whole scheme is non-rigorous except for linear functions, the error introduced by ignoring the higher powers of the corrections to the assumed values for the parameters or other unknowns does not seriously affect the result. It seems to be the basis for a very good graduate course and shows clearly how the use of Lagrange multipliers may sometimes clarify an otherwise hopeless analytical problem.

JOHN H. OGBURN

Les Definitions Modernes de la Dimension. By Georges Bouligand. Paris, Hermann, 1935. 44 pp.

This volume opens with a brief discussion of linear or affine geometry based on postulating the existence of an abstract system of elements called vectors which combine amongst themselves and with the real numbers to give us new elements of the same class. The dimension is determined by the number of linearly independent vectors. There is a brief discussion of spaces which in this sense would be of infinitely many dimensions.

Then denying the right of geometry to limit itself merely to spaces of this category, the author discusses three types of definition for dimension:

(1) The definitions which follow the direction of Fréchet who is interested in the spaces introduced by the needs of General Analysis. Fréchet develops a theory of dimension types or a topological frame based on the notion of homeomorphism between sets. His theory has as its axiomatic substructure three simple axioms concerning the closure of point sets.

(2) Those definitions where the fundamental notion is that of *separation* of sets by various subsets. These definitions emanate from the influence of Poincaré who started from the seeming contradiction of the existence of a (1-1) correspondence between the points of a line to which we should wish to assign the dimension one and the points of a plane which should be of dimension two. This line of development runs from Poincaré through Brouwer to Menger and Urysohn. The whole theory is based upon the three axioms fundamental in the Fréchet theory plus two additional axioms, that of normality and the second countability axiom, that is, a space in which distance can be defined. The author discusses the theorem of Lebesgue concerning intersection of coverings which can be used to give a definition of dimension independent of recurrence and which is usually used to show that the Menger-Urysohn definition gives the dimension n to sets to which we would intuitively assign the same dimension. A brief discussion is also given of the combinatorial point of view with respect to dimension which has been so ably developed by the powerful methods of Alexandroff.

(3) The definitions based on the notion of measure. The starting point here is the linear measure of Carathéodory, that is, measure of order unity. There is then given an outline of the generalizations due mainly to Hausdorff and the relation of this to the work on potential theory done mainly about the middle of the last decade. There is also a brief discussion of the work on transfinite diameter in which the names of Pólya, Szegö, and Fekete play the prominent role.

This book furnishes an excellent sketch of the various points of view and serves as an introduction to a more detailed study for which there are available Menger's book on dimension, Fréchet's *Espaces Abstraits*, and a host of original articles.

J. R. KLINE

Versicherungsmathematische Aufgabensammlung. Vol. 1. *Beiträge und Deckungsrücklagen in der Lebensversicherung.* By C. Boehm and E. Rose. 75 pp. Vol. 2. *Umwandlung von Lebensversicherungen.* By C. Boehm and P. Lorenz. 52 pp. Leipzig and Berlin, Teubner, 1937.

These two pamphlets contain a collection of problems with their solutions illustrative of the more simple types of calculations of an actuarial nature which are required in actual life insurance practice. Although the authors very properly point out in the two prefaces that no book can be a substitute for actual experience, these pamphlets are written from a more practical standpoint than most textbooks and provide an interesting and valuable insight into the practice of German life insurance companies. The first volume deals with the calculation of net single premiums, net annual premiums, gross premiums, net reserves, gross reserves, and special plans of insurance, in that order. The examples seem, on the whole, well chosen, and cover the ground well. The argument against the use of ultraconservative interest and mortality bases on page 12 seems to the reviewer rather naïve and unconvincing. On page 27 in a problem to determine what per cent of the gross premium the various elements of expense constitute, the collection cost is expressed as a percentage of the entire gross premium, while in the case of the clerical and administrative expense and the prorated initial expense the denominator used is the gross premium less the collection expense. This may be the German custom but would seem to call for a word of explanation. The statement on page 51 that the prospective formula is the simplest for calculating net reserves is true, in general, but it is odd that it should occur in the discussion of a special varying insurance for which the retrospective formula would have been easier.

The second volume gives the impression that the German treatment of the difficult problem of policy changes is characterized by the same balancing of theoretical accuracy against practical expediency which is typical of our own approach to the subject. The per mille symbol referred to by Professor Dodd in the September 1938 Bulletin in reviewing a similar book is used frequently, and by its similarity to the per cent sign may confuse the reader to whom it is not familiar. The explanations are clear, and, generally speaking, these books have accomplished their purpose in an admirable fashion.

T. N. E. GREVILLE

Stellar Dynamics. By W. M. Smart. Cambridge, University Press; New York, Macmillan, 1938. 8+434 pp.

This book is an effort to present, in considerable detail, the development of stellar dynamics from a mathematical treatment of the results of observations. The general theory of the correction of observed facts is treated and results of this treatment are applied to the correction of parallaxes, absolute magnitudes, and transverse velocities.

The parallaxes which are derived in Chapter VI by a statistical discussion of the proper motions are of especial interest, for it was by applying this method to the Cepheids that Shapley and others determined the distances of the globular clusters.

The mathematical theory of a single star drift and its conflict with observations led Eddington to the division of the stars into two drifts. Both the single and double drift theories are discussed, the former leading up to the determination of the solar motion and the latter connecting with Kapteyn's two stream theory. A later chapter develops Schwarzschild's ellipsoidal hypothesis, and the results obtained by the use of different theories are frequently compared.

Much of the work on stellar statistics, particularly that of a theoretical nature, has been available for twenty-five years, but the practical applications have been few due to the lack of sufficient observed data. The author supplies this want in Chapter VIII.

The last few chapters are largely concerned with the work of Jeans and Eddington on stellar dynamics, and some very recent results have been incorporated. One of the most interesting chapters, at least to the reviewer, is concerned with galactic rotation.

The entire book bristles with probability integrals and complicated mathematical formulas. One wishes that it might be possible to read such a book without encountering the probability integral so often. It is not clear, always, just what the author is trying to show nor what has been proved after a page of formulas. Perhaps this is unavoidable; at least the author warns us in the preface that he is aiming at a mathematical treatment. One wonders occasionally if his equations are not "yes men" who give a mathematical character and amplification to the ideas entrusted to them by their master.

The book is well worth the effort required to read it and is the best available account of the subject. Dr. Smart is to be congratulated on the successful completion of such a comprehensive and scholarly book.

H. E. BUCHANAN

Vectoranalysis. By Siegfried Valentiner. (Sammlung Göschen, no. 354.) Berlin and Leipzig, de Gruyter, 1938. 136 pp.

The first sixty pages are devoted to the definition, algebra, differentiation, and integration of vectors. To motivate the development the author draws freely upon mechanics. In the second part of the book vectors are applied to potential theory, hydrodynamics, and the theory of electricity. Part three deals with linear vector functions, dyads, and tensors with applications to the theory of elasticity. The book contains thirteen carefully chosen figures and closes with a table of the more important formulas used. The notation is conventional.

The task of including so much material in such a few small pages required skillful planning. Although the explanations are in general not detailed, the beginner will find the account readable. For the person already acquainted with the elements of vector analysis the book will be a useful handbook.

V. V. LATSHAW

Grundbegriffe und Hauptsätze der höheren Mathematik, insbesondere für Ingenieure und Naturforscher. By Gerhard Kowalewski. Berlin, de Gruyter, 1938. 156 pp.

Dr. Gerhard Kowalewski, finding that mathematics is taking a less and less prominent place in the German educational system, has felt obliged to do his part in presenting the fundamentals of higher mathematics "without which a profitable study of engineering and the natural sciences is inconceivable."

No one will hold it against Dr. Kowalewski that his very original 156 page booklet does not try to cover "the fundamentals and basic laws of higher mathematics" implied by the title. But it may be well to warn budding engineers and naturalists that this is no beginner's textbook.

Of the three chapters—"Vector Calculus and the Theory of Determinants," "Theory of Limits," "Differential and Integral Calculus"—the contents and the spirit of the last one came nearest to what one finds in the average textbook. The first chapter (an authoritative treatment of the relations between vectors and determinants) and the second one (Weierstrass' law and some of its conclusions) will be appreciated most by advanced students, and the place of these subjects in a condensed curriculum appears a little doubtful.

American readers will be tempted to compare Dr. Kowalewski's little volume with *Higher Mathematics for Engineers and Physicists* by I. S. and E. S. Sokolnikoff. Taking into account a 3 to 1 ratio in size in favor of the Sokolnikoffs, this reviewer believes that engineers will prefer the American work because of its excellent selection of important material and its "engineering approach."

Dr. Kowalewski's work, expert as it is, misses this appeal.

R. P. KROON

Your Chance to Win. The Laws of Chance and Probability. By H. C. Levinson. New York and Toronto, Farrar and Rinehart, 1939. 343 pp.

According to the advertisement of this book, the author "has taught mathematics at Ohio State University and has devoted more than eleven years to business statistics and to executive work in business." After reading the book, one wonders also how much time the author has spent at such places as Monte Carlo and Canfield's.

This book is written for the layman, so the mathematics involved is just an application of the laws of probability that are found in any college algebra. The topics covered by the book include such titles as luck, chance, statistics, the world of superstition, fallacies, heads or tails, poker chances, roulette, lotteries, craps, bridge, fallacies in statistics, statistics and science, and so on.

This book is more interesting for its practical psychology and common sense than for its mathematics. Everyone is superstitious and a "gambler at heart" (according to an old saying), so perhaps everyone should read this book. Again, mathematics is said to be just "organized common sense," so perhaps mathematicians should take also a scientific interest in this book.

At any rate, undergraduate mathematics clubs would find this book excellent for at least one meeting, if we may judge from the sudden and great interest shown by freshmen when the topic of probability is reached in the course in college algebra. The reviewer enjoyed immensely reading this book and was especially interested to find worked out why it is so hard to "fill the inside of a straight" in poker. However, a word of warning should be given at this point. The fundamental assumption of this book is that all the activities discussed therein are conducted honestly. As we all know, this assumption is so often not satisfied in practice.

ALAN D. CAMPBELL

Elementary Theory of Operational Mathematics. By Eugene Stephens. New York, McGraw-Hill, 1937. 11+313 pp.

This book is concerned primarily with the theory of symbolic operators and their application to the solution of differential equations. The book, as stated in the preface, "is an outgrowth of an attempt (1) to search out the history of these [operational]

methods; (2) to codify the set of theorems found; (3) to connect them with the work of the rigorists; and (4) to extend the theorems by all possible means." It seems to the reviewer that the author has succeeded well in his attempt.

The nature of the book is indicated by the chapter headings: I, Introduction and Definitions (8 pages); II, The Operator $D \equiv d/dx$ (47 pages); III, Applications to Ordinary Linear Differential Equations with Constant Coefficients (14 pages); IV, Algebraic Theorems (determinants, 18 pages); V, Matrices (12 pages); VI, Systems of Ordinary Linear Differential Equations with Constant Coefficients (26 pages); VII, The Operators $d_1 \equiv \partial/\partial x$, $d_2 \equiv \partial/\partial y$ (30 pages); VIII, Applications to Partial Linear Differential Equations with Constant Coefficients (18 pages); IX, The Operator $d_i \equiv \partial/\partial x_i$ (6 pages); X, The Noncommutative Operator $xD \equiv \theta$ (8 pages); XI, Solutions in Series (41 pages); XII, The Differential Equation in Mathematical Physics (9 pages); XIII, Initial or Terminal Conditions (10 pages).

The last two chapters belong more properly in a text on differential equations and are so excellent that they should form the introduction to every beginning course in that subject. The first explains the nature of a differential equation and its solution, and gives the most illuminating discussion of these matters that the reviewer has seen in any book. The second shows the meaning of the constants of integration and how to determine them in a wide variety of problems.

The book concludes with three appendices and an index. The third appendix gives the history of operational mathematics and a complete bibliography of the subject from 1765 to the present time.

J. B. SCARBOROUGH

An Introduction to the Theory of Numbers. By G. H. Hardy and E. M. Wright. Oxford, Clarendon, 1938. 16+403 pp.

As the authors have taken pains to describe—too modestly—the nature of their work, we quote from their preface.

"This book has developed gradually from lectures delivered in a number of universities during the last ten years, and, like many books which have grown out of lectures, it has no very definite plan.

"It is not in any sense (as an expert can see by reading the table of contents) a systematic treatise on the theory of numbers. It does not even contain a fully reasoned account of any one side of that many-sided theory, but is an introduction, or a series of introductions, to almost all of these sides in turn. . . . There is plenty of variety in our programme, but very little depth; it is impossible, in 400 pages, to treat any of these many topics at all profoundly."

Those who had the pleasure of hearing the senior author's lectures when he was in the United States ten years ago, will have pleasurable anticipations of what to expect; nor will they be disappointed. The book is like no other that was ever written on the theory of numbers, as an introduction or as a treatise; although Edouard Lucas might have written something like it had he been primarily interested in the analytic theory and were he living today. Some of the topics treated have been frequently discussed in the English and German journals of about the past decade. As might be anticipated from the authors' interests, analysis dominates much of the material. The treatment throughout, even of old things, is fresh and individual.

Owing to the widely varied character of the matter, it is impossible to give a brief summary of the scope of the book, and the following sample must suffice to indicate the contents. The theory of quadratic forms is omitted. Chapters 1, 2 treat the series of primes and the fundamental theorems on divisibility for the rational integers.

Enough of the O, o, $\sim$ notation is explained for the statement (page 9) of the prime number theorem. Solved and unsolved problems on primes are discussed, including what the authors call "unreasonable" conjectures. Chapter 3 is on Farey series and Minkowski's theorem for lattice points within convex contours. In somewhat the same order of ideas, Chapter 23 proves Kronecker's theorem (1887) for n linearly independent irrationals by three methods due respectively to Lettenmeyer, Estermann, and H. Bohr, of which the first is semi-geometrical, the second inductive, and the third analytic. The Minkowski theorems are those on linear forms (the non-homogeneous case included) which appear in the theory of units in algebraic number fields. Attention is called (page 398) to an outstanding unverified conjecture of Minkowski concerning a conceivable generalization of his theorems.

Having mentioned algebraic numbers, we note next what the book contains about them. First, historians will read with interest the digressions on pages 42–45, 180–181. (Incidentally, was Gauss the first to prove, with a slight gap in one lemma, the fundamental theorem?) Chapter 4 is on quadratic irrationals, and includes proofs of the irrationality of e, e^2. Algebraic numbers are defined (Chapter 11) in connection with approximation of irrationals by rationals, and Cantor's theorem on the denumerability of the set of all algebraic numbers is proved; Liouville's construction of transcendentals is given; and finally, the transcendence of e, π is shortly proved, substantially as by Landau. More traditional material on algebraic numbers appears in Chapters 14, 15, where units, primes, and the fundamental theorem are first discussed for certain quadratic fields of class number unity, leading up to the simplest classical examples for which the fundamental theorem fails. Euclidean quadratic fields (those in which a G.C.D. process holds) have recently received much attention; it is proved that the number of real quadratic euclidean fields is finite. An uncommon application of algebraic fields is Western's proof (page 223) of a primality test due to Lucas. Ideals are defined for quadratic fields and are illustrated in one usual way by point-lattices, but their theory is not developed.

Under congruences and allied topics (Chapters 5–8), in addition to the customary material of an introduction, some more recent novelties are included, such as the divisibility of $2^{p-1}-1$ by p^2, p prime, factors of Mersenne numbers, Wolstenholme's (Waring's) theorem, an inductive proof of the Clausen-von Staudt theorem, Bauer's identical congruence, and applications of Euler's ϕ-function to Gauss and Ramanujan sums. Without attempting an exposition of the Gaussian theory of cyclotomy, the authors give Richmond's construction for the regular 17-gon, with a slight suggestion making "it plain to the reader (as is plausible from the beginning)" why the synthetic magic works. Obviously a good deal of the material in these chapters, as in most of the others, is intended to soften the shock some readers may be expected to experience if and when they follow up these suggestive introductions by systematic study.

Some of the simplest arithmetical functions are discussed in connection with the classic inversion formulas in Chapter 16. Opportunity is taken to prove Euclid's theorem on perfect numbers. Chapter 17 on generating functions of arithmetical functions gives a brief introduction to the customary topics under this head. The following chapter discusses the order of certain of the simpler arithmetical functions —number and sum of divisors, Euler's function, the number of lattice points in a circle, without excessive refinements of the analysis. This is followed (Chapter 19) by a sketch of the theory of partitions as a typical problem in additive arithmetic, enough of Jacobi's identity being proved to yield some of the more interesting classical theorems. Graphical proofs are also given, and one of Ramanujan's congruences is proved. The chapter concludes with the proofs of two Rogers-Ramanujan identities and a hint of the related continued fractions.

Chapter 20 on representations as sums of 2 or 4 rational integer squares provides an opportunity to sketch the arithmetic of quaternions, and serves as an introduction to Waring's problem for cubes and fourth powers in Chapter 21, where Tarry's problems, sums $\sum \pm x_i^k$, and equal sums of two kth powers are also briefly considered. Any reader of this notice who is looking for something hard to do is invited to observe page 336, (f). The notes on pages 334–337 summarize the main results up to 1938 on Waring's problem.

Chapter 22, the third in the book on the series of primes, follows Landau's proofs of Tchebychef's theorems, and pushes the analysis a little further to obtain Mertens' approximation to $\prod_{p \leq x} (1-p^{-1})$, p prime. The chapter concludes with a note on "round" numbers and a theorem of the "almost all" type on the order of the number of divisors of n.

As the last of the authors' introductions, we mention Chapter 13 on certain Diophantine equations ($x^n+y^n=z^n$, $n=2, 3, 4$), the expression of m as a sum of rational cubes, $x^3+y^3=3z^3$, and equal sums of two cubes.

The foregoing sample from the two dozen chapters covering 400 pages may give some idea of the extraordinary richness of the material, and suggest the justice of the authors' own characterization of their work as "a series of introductions" to a vast and many-sided theory. They have presented these introductions in a manner that should stimulate a reader to continue beyond some of them; and it seems safe to say a great deal more than what they themselves say, "we can hardly have failed completely, the subject-matter being so attractive that only extravagant incompetence could make it dull." The book is anything but dull; in fact it is as lively as the proverbial (not the English) cricket.

E. T. Bell

Le Concours de l'Analyse Mathématique à l'Analyse Expérimentale des Faits Statistiques. By Georges Hostelet. (Actualités Scientifiques et Industrielles, no. 585; Le Progrès de l'Esprit; Exposés publiés sous la direction de L. Brunschvicg.) Paris, Hermann, 1937. 70 pp.

The first section can be summed up by stating that the experimenter believes that the basic laws on which statistics are founded have been proven by the mathematician, while the mathematician believes that these laws have been demonstrated by the experimenter.

In the next section the author states that the mathematical analysis of statistical facts are contained in the intuitive-empirical notion of species, which implies notions of means, errors of means, frequencies and limits of these errors. He gives the conditions, which a statistical fact must satisfy in order that the arithmetic mean signifies the best value, defines the analytic and synthetic indicator of the degree of correlation, outlines the conditions necessary for linear correlation and considers that mathematical analyses are auxiliaries to experimental analyses of measurable facts when measurements present numerical relations expressible in explicit functions.

Section three contains postulates on which mathematical and experimental reasoning depend, answers to objections Fréchet made concerning probability and its use in the analysis of statistical facts, the necessary steps for verifying an hypothesis and an explanation of the difference between mathematical and experimental reasoning.

Chapter IV presents the three modes of scientific investigation, viz: the empirical-intuitive mode, the deductive-abstract mode and the abstract-experimental mode and points out their limitations.

The following chapter contains discussions pertaining to philosophy, philosophic and scientific certitude, philosophic and scientific attitudes, types of judgments and modes of thought used in reasoning, and comparisons between social, physical and mathematical sciences. The author states that his efforts, by experimental analysis of observations of statistical facts, have been to disengage their characteristics and their correlations and to establish the limitative conditions of the experimental validity concerning them. In order to reveal the danger of the abstract reasoning, which menaces the course of mathematical analyses in the experimental analyses, he insists on the existence of the conditions which should be realized in order that the formulas, obtained from mathematics, be legitimate.

In the first chapter of Part II the author states the ideas of Reichenbach concerning causality, uncertainty of scientific propositions and predicting future events. He compares his views with those of Reichenbach, examines the notion of uncertainty in science, states that the idea connected by Reichenbach to the idea of uncertainty is associated with the absence of causality, shows how causality is connected with probability and discusses the relations of philosophy to scientific procedure of experimental analysis.

Chapter VII contains de Broglie's definition of determinism, an explanation of how de Broglie considers causality and determinism and his illustration relating to electrons shot from a cannon, bombarding a surface of a crystal, quotations from Planck, Einstein, Lord Rutherford relating to causality, a discussion of philosophic and scientific determinism in relation to causality and de Broglie's views on probability and frequency.

The following chapter presents the position of Barzin on notions of causality and determinism, which are characteristic of the attitude of logicians. Logic researches concerning probability establish the necessity of affirming a principle of induction in the sense that it adopts universal determinism; each theory of probability implies logically causal relations. By the use of perfect and imperfect games of chance the author compares his views with those of Barzin and points out the difference between the calculus of probability and statistical laws. Barzin believes that the principle of statistical induction rests on the affirmation of a determinism; Hostelet believes that this principle implies causality.

The last two chapters treat of comparisons of the philosophic and scientific attitude and the role of scientific methodology. Near the close of the book the author states that to practice the scientific method is to give the mind more security in the acquisition of knowledge, is to increase the surety and the power of its faculties of intuition and invention, is to prepare the accord of intelligences to render them tolerant and conciliatory.

W. D. BATEN

Science in a Tavern. By Charles S. Slichter. Madison, Wisconsin, University Press, 1938. 9+186 pp.

The title of this book refers to the meetings in London taverns and private houses, beginning about 1650, of English scientists and men of means and philosophical interests. At these meetings a great deal was eaten and drunk, discussion and even experimentation took place. It was from this nucleus that the Royal Society was formed in 1662. After that an inner circle still dined before the meetings of the Society and in 1725 formed a dining club. Dean Slichter, in the first two essays of this volume sketches the parallel early histories of the Royal Society and of the Club, stressing the importance for science of the patronage of the men of wealth and social position who belonged to these organizations.

The following essay deals with men of universal genius, polymaths they are called; Archimedes, Leonardo, Euler, the three Bernoullis, John Wallis, Sir Christopher Wren, Newton, Thomas Young, Lagrange, Hamilton, D'Alembert, Huygens are briefly mentioned as examples. The careers of two polymaths, Kelvin and Heaviside, are discussed in more detail and an interesting comparison is drawn between the fame of Kelvin during his lifetime and the comparative obscurity of Heaviside. Kelvin is described as an extrovert, Heaviside as an introvert with greater depth both scientific and spiritual. In another essay, on Newton, credit is given him for industrialism: "If recent industrialism is a blessing, give initial credit to the *Principia.*" This oversimplification reflects the general tendency of the book to discuss only English scientists and contributions to science.

In an essay entitled "Industrialism" Dean Slichter defends the scientific method as capable of resolving the conflicts of modern civilization. "Science brings its own remedies and removes the evils that it has itself created. If it were otherwise science would not be science." He evidently does not take the point of view that science assists men to get what they want, but must leave the determination of those wants to factors outside the domain of science. He assumes: "There is a best way and experts are selected to find and direct it." He believes, apparently, in "government for the people." The essay entitled "The New Philosophy" discusses controlling the power provided by science. Dean Slichter says: "in England spiritual control may grow and spread from the Universities. In America I expect the hope of the new philosophy to lie not with the university faculties, but with men of the world; with leaders in the industries; with engineers and business men and lawyers and men close to affairs. We must look for a new Christopher Wren who can look upon life as a whole. . . ."

W. FLEXNER

Essai sur l'Unité des Sciences Mathématiques dans leur Développement Actuel. By Albert Lautman. (Actualités Scientifiques et Industrielles, no. 589.) Paris, Hermann, 1938. 62 pp.

In the introduction to the first edition of his *Gruppentheorie und Quantenmechanik*, Hermann Weyl remarks that whereas it was fashionable in the past century to arithmetize all branches of mathematics (for example, the study of geometry was reduced to the study of a metric), it has now become fashionable to axiomatize all branches of mathematics (the study of analysis is now based on a study of abstract spaces). Lautman attaches undue significance to these remarks, and interprets them as meaning that there is a schism in mathematics. Lautman's personal belief is that the distinction between the two kinds of mathematics corresponds only to historical conditions in the development of mathematics, and he undertakes to prove in this book that the schism implied by Weyl's remarks does not exist.

As a matter of fact, no such schism exists, and it is to be doubted if any mathematician really thinks that one does exist. A careful reading of Weyl's remark in its context would seem to indicate only that he was drawing an analogy between the earlier change in the point of view of the mathematicians and the change then occurring in the point of view of the physicists because of the new quantum mechanics. So Lautman is tilting against windmills.

In spite of its inconsequential result, the book is interesting to read. Lautman takes algebra (of the van der Waerden type) and topology as representative of the one sort of mathematics, and analysis as representative of the other sort of mathematics. He then gives many examples of cases where the methods or results of one are used in the other. In the first chapter he discusses uses of the ideas of linear dependence and dimensionality in analysis, as in Hilbert space for instance. Also he brings out

some less direct analogies, such as exist between the Weierstrass factorization theorem and the factorization of polynomials.

The second chapter discusses the uses of non-euclidean metrics in analysis. This seems a bit irrelevant.

The third chapter calls attention to the extensive use of noncommutative algebras in connection with Lie groups, differential operators, and so on.

The last chapter concerns the analytic theory of numbers. This subject is particularly relevant because of the constant interplay of results from number theory and analysis.

BARKLEY ROSSER

Mécanique des Fluides. By Joseph Pérès. Paris, Gauthier-Villars, 1936. 8+322 pp.

It is not surprising that hydromechanics has for many years attracted the serious efforts of eminent mathematicians, such, for example, as Newton, Euler, Lagrange, Cauchy, Poincaré, Levi-Civita, to name only a few. For, both in the formulation of its fundamental concepts and in its methods, hydromechanics is essentially a part of mathematics. It postulates a "mathematical fluid" with certain properties, and in order to deduce further properties it calls on a wide range of mathematical doctrines, potential and function theory, differential equations, calculus of variations, and so on.

During the present century the researches of Oseen and Prandtl have done much to narrow the gap between theory and experiment. More recently the urgent needs of the science of flight have furnished a fertile soil which already supports a luxuriant theoretical growth.

The purpose and general character of the book under review may be seen from the following quotation from the preface by Henri Villat, who is generally regarded as the leading spirit in the study of hydromechanics in France. "The principal object of these lessons (at the Sorbonne) is the mathematical explanation of the resistance of fluids, and in particular the theory of the lifting wing." As indicated by this quotation the emphasis is on the force experienced by a body placed in a uniform stream. The following outline will describe the contents (the numbers refer to chapters).

A. Theoretical hydromechanics.
   1– 3 Mechanics of perfect fluids.
   4– 7 Two-dimensional motion (complex variables), flow past obstacles, profiles of Joukowski and others, formulas of Kutta and Blasius.
   9–10 Three-dimensional motion with vorticity.

B. The problem of resistance.
   8 Discontinuous motion, work of Villat and others.
   11 The Prandtl wing theory.
   12 The method of Oseen.

The reader unacquainted with the subject will find in Chapters 1–3 a clear introduction. Chapters 4–7 treat the topics covered in considerable detail. These three chapters, along with Chapter 9, will appeal to those interested in aerodynamics. The latter chapter is especially recommended for its neat presentation. Only in the last chapter is viscosity directly taken into account. But the reader who expects to find in the twenty-nine pages of this chapter an easy introduction to the Oseen theory is likely to be disappointed. The treatment is quite novel and at places difficult to follow for one not already acquainted with the theory.

The author's presentation is clear and direct and at times ingenious. He has achieved a number of refinements and simplifications which make the book worth

the study of those well acquainted with the subject. The book will take its place among the important treatises on hydromechanics.

C. A. SHOOK

Gesammelte Abhandlungen. Geometrische Abhandlungen. By Sophus Lie Edited by Friedrich Engel and Poul Heegaard. Vol. 1, 1934. 862 pp. Vol. 2, 1937. 506 pp. Leipzig, Teubner; Oslo, Aschehoug

These two volumes contain Lie's publications on geometry. We have Lie's first papers on imaginary plane geometry and line geometry. Already in this early work the consideration of transformation groups and their relations to geometry which are predominant in his later work can be found. The general trend of Lie's way of thinking can best be studied in the numerous memoirs on contact transformations and minimal surfaces. Characteristic of the beauty of Lie's papers on geometry are the articles dealing with the connection between translation surfaces and the theory of abelian functions. In these two volumes we find many papers in which the foundations for the modern theory of Pfaffian systems and general geometry were given. The collected works proper are augmented by two commentaries of the editors. These commentaries give many critical and historical remarks which are extremely useful for the study of the various papers.

O. F. G. SCHILLING

NOTES

The University of Notre Dame has begun the publication of a series entitled Reports of a Mathematical Colloquium, edited by Professor Karl Menger. The reports are to form a new series following Ergebnisse eines mathematischen Kolloquiums which was published in Vienna from 1928 to 1936. The first issue, designated as second series, issue 1, has recently appeared.

The establishment of the Iowa State College Press, a publishing outlet for writings of book length in the fields of science and technology, has been announced.

The French Association for the Advancement of Science will meet at Liège on July 17–22, 1939.

The Sixth Pacific Science Congress will be held in Berkeley, Stanford, and San Francisco, California, from July 24 to August 12, under the auspices of the National Research Council.

The François Deruyts Prize of the Royal Academy of Belgium has been awarded to P. Burniat of the University of Brussels and F. Rozet of the University of Liège.

Professor W. D. Cairns, Secretary-Treasurer of the Mathematical Association of America, has announced that the Department of Mathematics of Brooklyn College, Brooklyn, New York, has won the first prize of $500 in the second annual William Lowell Putnam Mathematical Competition, held March 4, 1939. The members of the winning team were Richard Bellman, Abraham Hillman, Bernard Sherman. The second prize of $300 is awarded to the Department of Mathematics of Massachusetts Institute of Technology, Cambridge, Massachusetts, members of whose team were R. P. Feynman, J. W. Follin, Jr., H. E. Singleton. The third prize of $200 is awarded to the Department of Mathematics of Mississippi Woman's College, Hattiesburg, Mississippi, members of whose team were Nina P. Byrd, Mary E. Fancher, Ethel L. Tate. In addition to these prizes to the departments of mathematics with winning teams, a prize of $50 each is awarded to the following five persons whose scores ranked highest in the six-hour examination (names arranged alphabetically): R. P. Feynman, Massachusetts Institute of Technology; Abraham Hillman, Brooklyn College; E. L. Kaplan, Carnegie Institute of Technology; William Nierenberg, College of the City of New York; Bernard Sherman, Brooklyn College. One of these five will receive a $1000 year scholarship at Harvard University, this award to be announced later. The members of the three winning teams will receive individual prizes of $50, $30, and $20, according to the ranks of their teams, and all individuals receiving prizes will also receive medals. Honorable mention has been awarded this year to three teams and to seven individuals. The teams are from the Department of Mathematics, College of the City of New York, members being Herbert Mintzer, William Nierenberg, Harry Soodak; the Department of Mathematics, Cooper Union Institute of Technology, New York, members being Theodore Berlin, Benjamin Lax, Samuel Manson; and the Department of Mathematics, University of California, Berkeley, members being W. M. Kincaid, C. W. Lippmann, S. A. Schaaf. The seven individuals receiving honorable mention are: Richmond Albert and Richard Bellman, Brooklyn College; Theodore Berlin and Benjamin Lax, Cooper Union Institute of Technology; C. W. Lippmann, University of California; M. J. Norris, College of St. Thomas; T. S. Schreiber, Johns Hopkins University.

The Franklin Institute has awarded the Elliott Cresson Medal to G. A. Campbell of the American Telephone and Telegraph Company, to J. R. Carson of the Bell Telephone Laboratories, and to C. V. Boys of London.

The John L. Lewis Prize of the American Philosophical Society has been awarded to Professor H. N. Russell of Princeton University.

On May 18, 1939, the seventieth birthday of Professor Élie Cartan was celebrated in Paris with appropriate ceremonies. A commemorative volume of his works is being published. A volume of the Journal de Mathématiques will be dedicated to him.

Professor A. Denjoy of the University of Paris has been elected an honorary member of the American Academy of Arts and Sciences.

Mr. M. H. A. Newman, lecturer in mathematics, Cambridge, has been elected a fellow of the Royal Society of London.

Professors F. Severi and E. Bompiani have been named members of the editorial committee of the Zentralblatt für Mathematik.

Professor G. D. Birkhoff of Harvard University has been appointed exchange professor to France for the second half of the academic year 1939–1940.

Professor William Marshall of Purdue University has retired with the title emeritus.

Professor J. H. Van Vleck of Harvard University gave a series of lectures at the Institute Henri Poincaré at the University of Paris during the month of May. He also took part in a symposium on magnetism at Strasbourg.

Professor Oscar Zariski of Johns Hopkins University has been awarded a John Simon Guggenheim Fellowship.

Dr. Leon Alaoglu, Dr. D. T. Perkins, and Dr. B. J. Pettis have been appointed Benjamin Peirce instructors at Harvard University for the academic year 1939–1940.

Professor Silvio Cinquine has been appointed to a professorship at the University of Pavia.

Professor Basilio Manià of the University of Pavia has been appointed to a professorship at the University of Milan.

Professor R. Rothe of the Technical School of Berlin has retired.

Professor Antonio Signorini of the University of Naples has been appointed to a professorship at the University of Rome.

Professor Leonida Tonelli of the University of Pisa has been appointed to a professorship at the University of Rome.

Dean Daniel Buchanan of the University of British Columbia will be visiting professor of astronomy and mathematics in the summer session at Los Angeles from June 26 to August 4.

Assistant Professor Jewell Hughes Bushey of Hunter College has been promoted to an associate professorship.

Professor L. E. Dickson of the University of Chicago has announced his retirement, effective at the end of this academic year.

Assistant Professor C. H. Forsyth of Dartmouth College has been promoted to a professorship.

Associate Professor L. M. Graves of the University of Chicago has been promoted to a professorship.

Dr. Arthur Ollivier of Mississippi State College has been promoted to an assistant professorship.

Assistant Professor R. L. O'Quinn of Louisiana State University has been promoted to an associate professorship.

Dr. N. E. Steenrod of Princeton University has been appointed to an assistant professorship at the University of Chicago.

Professor A. H. Wilson of Haverford College has retired.

Professor D. R. Curtiss of Northwestern University has been on leave of absence for the second semester of 1938–1939.

The following appointments to instructorships are announced: Armour Institute of Technology: Dr. W. T. Scott; Bradley Polytechnic Institute: Mr. R. N. Johanson; University of Chicago: Dr. O. F. G. Schilling.

Professor Emeritus C. M. Jessop of the University of Durham died on March 9, 1939, at the age of seventy-six years.

The death of Professor Reinhold Müller of the Technical School of Darmstadt has been reported.

The death of Georges Urbain on November 5, 1938, has been reported.

Mr. J. C. Rietz, vice president and actuary of the Midland Mutual Life Insurance Company of Columbus, Ohio, died April 21, 1939. He had been a member of the Society since 1921.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

259. E. E. Betz: *On accessibility and separation by simple closed curves.*

In this note it is shown that if M is a Peano continuum such that for every simple closed curve J in M the set $M-J$ consists of at least two and at most a finite number of components, then every boundary point of each such component D is regularly accessible from D. This theorem leads to new characterizations of the simple closed surface. (Received April 6, 1939.)

260. Garrett Birkhoff: *Lattice theory of Brouwerian logic.*

Brouwerian logics can be defined as lattices with 0 and I and an implication operation $x \rightarrow y$ which satisfies (1) $x \rightarrow (y \rightarrow z) = (x \cup y) \rightarrow z$, (2) $x \rightarrow y = 0$ if and only if $x \geqq y$. It can then be proved that no lattice will support more than one implication operation making it into a Brouwerian logic, and that a lattice will support as many as one such implication operation if and only if it satisfies a certan non-self-dual infinite distributive law. (Received April 10, 1939.)

261. R. P. Boas (National Research Fellow): *Integral functions bounded on the real axis.*

Let $f(z)$ be an integral function of order one and type $k < \pi$. If the numbers $f(\lambda_n)$ are bounded, where the λ_n are real, and $|\lambda_n - n| \leqq 1/(2\pi^2)$, $(n = 0, \pm 1, \pm 2, \cdots)$, then $f(x)$ is bounded for real x. The proof involves the Paley-Wiener theory of non-harmonic Fourier series. When $\lambda_n = n$, the theorem reduces to a theorem of Miss Cartwright, for which a new proof is incidentally obtained. (Received April 15, 1939.)

262. R. H. Cameron: *Extensions of Wiener's general Tauberian theorem.*

If $f(x)$ is bounded and measurable and its faltung with each function $g(x)$ of L approaches $A\int g(x)dx$ as $x \rightarrow \infty$, we say that wlim $f(x) = A$, the limits of all integrals being $-\infty$ to ∞. Then one has the theorems: (1) Let $g(x)$ be a function of bounded variation such that $\int e(tx)dg(x)$ never vanishes. Then if $f(x)$ is bounded, Lebesgue measurable, and Radon measurable with respect to $g(t-\lambda)$ for all λ, and if $\int f(x-t)dg(t) \rightarrow A\int dg(t)$ as $x \rightarrow \infty$, it follows that wlim $f(x) = A$. (2) Let $g(x)$ be a function of bounded variation whose discrete and singular parts are $h(x)$ and $s(x)$; and let lower bound $\left|\int e(tx)dg(t)\right| > 0$ and lower bound $\left|\int e(tx)dh(t)\right| > \int |ds(t)|$. Then if $f(x)$ is bounded and Radon measurable, $\int f(x-t)ds(t)$ is a Baire function, and $\int f(x-t)dg(t) \rightarrow A\int dg(t)$, then it follows that lim $f(x) = A$. In this case the limit is an ordinary (strong) limit. (Received April 20, 1939.)

263. C. R. Cassity: *Two theorems on the rational quartic surface in space of four dimensions.* Preliminary report.

The first of these theorems gives explicit equations for the cones in the pencil of quadrics on the quartic surface in terms of the coordinates of the fundamental points of the plane representation. These equations are such that all members of the pencil are written as sums of squares. The second theorem relates the elliptic and hyperbolic points of the surface as defined by Fabricius-Bjerre (Mathematische Zeitschrift, vol. 41 (1936), pp. 686–707) to the map on the surface as studied by the author in an article soon to appear in the Duke Mathematical Journal. (Received April 1, 1939.)

264. J. J. DeCicco: *The differential geometry of the curves of the Kasner plane.*

This is a continuation of two papers by Kasner (Science, vol. 85 (1937), pp. 480–482; Proceedings of the National Academy of Sciences, vol. 23 (1937), pp. 337–341), abstract 42-11-398 by Kasner, and abstracts 44-11-444 and 45-1-83 by the author. This paper studies the differential geometry of the curves of the Kasner plane K_3 with respect to the fundamental three-parameter group G_3, which is induced by the group of direct conformal transformations. The limit R of the ratio of the arc of any curve to its chord is found to consist of all rational numbers R such that R^2-R is the square of a rational number. If at every point of a curve the number R is unity, the curve is said to be a general curve. For a general curve $y=y(x)$, the length of arc is given by $s=\int y'^{-1}dx$ and its curvature K is given by $y''/2$. The intrinsic equation of a general curve is $K=K(s)$ where K is a function of s. We find the necessary and sufficient condition that ∞^1 parabolic-circles be a set of osculating parabolic-circles. The theory of evolutes and involutes is developed. Finally the differential geometry of fields of lineal elements of this plane is briefly considered. (Received April 24, 1939.)

265. J. J. DeCicco: *The conics of the Kasner plane.*

This paper studies the geometry of the conics of the Kasner plane with respect to the Kasner group G_3. The locus of all points of this plane which satisfy a quadratic equation is called a conic. The conics may be classified into twelve distinct types with respect to the group G_3. The number of invariants of each type is 0, 1, 2. A geometric construction for the conics is found and a geometric interpretation for each of the invariants. A general conic is the locus of a point such that the ratio $\epsilon \neq 0$ of its distance from a fixed point (the focus) to its distance from a fixed general line (the directrix) is constant. The general conic is an ellipse, parabola, or hyperbola according as $0<\epsilon<1$, $\epsilon=1$, or $\epsilon>1$ or $\epsilon<0$. A general central conic has two foci which lie on the conic and two parallel directrices. The minor axis of a general conic is orthogonal (dihorn angle 1/2) to the directrices. Finally a general central conic is the locus of a point such that the sum of its distances from two fixed points (the foci) is equal to a nonzero constant. (Received April 24, 1939.)

266. R. P. Dilworth: *A characterization of complemented modular lattices.*

A lattice Σ is said to be (1) an exchange lattice of type II if $(a, b)>a$ implies $b>[a, b]$, (2) a Jordan lattice if all maximal chains joining two given elements have the same length, (3) relatively complemented if $a \supset b$ implies there exists b' such that $(b, b')=a$, $[b, b']=z$ where z is the null element of Σ. K. Husimi has conjectured that a relatively complemented lattice Σ is modular if every relatively complemented

sublattice is a Jordan lattice. It is shown here that the theorem in the above form does not hold but that a lattice satisfying the hypotheses of the theorem is an exchange lattice of type II. Hence if the dual hypothesis is added to the theorem, the conclusion that Σ is modular is correct. The following theorem is also proved: Every archimedean complemented, non-modular lattice with unit element i and null element z has a sublattice $\{i, a, b, c, z\}$ where $a \supset c$, $(b, c) = i$ and $[a, b] = z$. As an immediate corollary we have the theorem due to G. Birkhoff and M. Ward (abstract 39-1-78) that a lattice of finite dimensions is a Boolean algebra if and only if every element of the lattice has a unique complement. (Received May 1, 1939.)

267. R. P. Dilworth: *Note on the prime elements of a modular lattice.*

An element p of modular lattice is said to be prime if $p \supset [a, b]$ implies $p \supset a$ or $p \supset b$. An irreducible q is said to be isolated if when it occurs in the reduced representation of an element as a crosscut of irreducibles it occurs in every such representation. It is then shown that an element p of a modular lattice in which the ascending chain condition holds is a prime if and only if it is an isolated irreducible. It is also shown that a lattice with chain condition is distributive if and only if every irreducible is a prime. As an application of these two theorems we have the result due to G. Birkhoff that a modular lattice in which every element has a unique representation as a reduced crosscut of irreducibles is distributive. (Received May 1, 1939.)

268. D. W. Hall (National Research Fellow): *On new characterizations of the 2-sphere.* Preliminary report.

In this note an accessibility theorem of E. E. Betz (see abstract 45-7-259) is employed to demonstrate the following theorem: If M is a compact continuum separated by no pair of its points but such that there exists an integer N such that for every simple closed curve J in M the set $M-J$ has at least two and at most N components, then M is a 2-dimensional sphere. The case $N=2$ gives a known theorem of Leo Zippin. Other characterizations are also obtained. (Received April 7, 1939.)

269. M. R. Hestenes: *On the first necessary condition for minima of double integrals.*

For a simple double integral problem in the calculus of variations the first variation of the double integral is of the form $L(\eta) = \iint_A (u\eta_x + v\eta_y + w\eta)dxdy$. It is assumed that A is a region in xy-space whose boundary C is composed of a finite number of disjoint simply closed regular arcs and that $A+C$ can be divided into a finite number of parts bounded by simply closed regular arcs on each of which the functions u, v, w are continuous. In the present paper it is shown that if $L(\eta)=0$ for every admissible variation η having $\eta \equiv 0$ on C, then $L(\eta) = \int_C \eta(udy - vdx)$ for every admissible variation η. Setting $\eta \equiv 1$ one obtains the Coral-Haar equations. The proof is direct and does not involve the use of the fundamental lemma for simple or double integrals. Further consequences of this result are given. (Received April 14, 1939.)

270. E. R. Lorch: *Means of iterated transformations in reflexive vector spaces.*

The paper gives a proof of the following "mean ergodic theorem": Let V be a bounded linear transformation in a reflexive vector space $\mathfrak{B}$ whose iterates V^n, $n=0, 1, 2, \cdots$, are uniformly bounded, $|V^n| \leqq K$. Then the means $1/n \sum_{i=0}^{n-1} V^i$ converge strongly to a limiting transformation which is a projection P ($P^2 = P$),

with $|P| \leqq K$. $Pf=f$ if and only if $Vf=f$. $Pf=0$ if and only if f is in the closure of the range of $V-I$ ($I=V^0$). The theorem includes as special cases the results of F. Riesz as well as their recent generalization (for the case $K \leqq 1$) by Garrett Birkhoff. The following theorem is decisive in the proof: Let $\mathfrak{B}$ be reflexive, T a bounded linear transformation in $\mathfrak{B}$. Let $\mathfrak{M} \subset \mathfrak{B}$ be the manifold on which $Tf=0$, $\mathfrak{N}$ be the closure of the range of T. Let $\overline{T}$ be the adjoint of T defined over $(\mathfrak{B})$, the adjoint space of $\mathfrak{B}$; let $(\mathfrak{M})$ and $(\mathfrak{N})$ be defined for $\overline{T}$ as indicated. Then $(\mathfrak{M})=\mathfrak{N}^{\perp}$, the orthogonal complement of $\mathfrak{N}$, $(\mathfrak{N})=\mathfrak{M}^{\perp}$, $\mathfrak{M}=(\mathfrak{N})^{\perp}$, and $\mathfrak{N}=(\mathfrak{M})^{\perp}$. (Received April 10, 1939.)

271. A. D. Michal and A. B. Mewborn: *General flat projective geometry.*

In this study the authors first generalize the notion of a flat projective space of Veblen and Whitehead (Cambridge Tracts in Mathematics and Mathematical Physics, no. 29, 1932) by replacing their arithmetic space of projective coordinates with a Banach space B_1 of couples (x, x^0) as projective coordinates, where x is in a Banach space B of allowable coordinates and x^0 is a real variable. It is then shown that any transformation of coordinates from allowable coordinates in B to general projective coordinates in B_1 satisfies a completely integrable abstract second order differential equation. A new existence theorem for a system of completely integrable abstract first order differential equations is then proved. This theorem is applied to the characterization of a flat projective geometry as a subclass of general curved projective geometry treated in the previous paper of the authors (*General projective differential geometry of paths*). (Received April 15, 1939.)

272. A. N. Milgram: *Partially ordered sets: bounds and mappings.*

We consider a partially ordered set A and a lowering function $l(a) \leqq a$ defined on A Sufficient conditions are given in order that there exist an element a such that $l(a)=a$. The subset U of A is called an upper section if $a<b$ and $a \in U$ imply $b \in U$. A system of upper sections σ with the property that $a<b$ implies there is an element $U \in \sigma$ such that $a \notin U$ and $b \in U$ is called a separating system of A. If each well-ordered decreasing sequence of power at most P has a lower bound in A, and if A has a separating system of power P, then there is always an element a such that $l(a)=a$. This depends on the fact that no well-ordered decreasing sequence of elements of A can have a power greater than P. In the second section a type of universal ordered set is given into which all partially ordered sets may be mapped so that $a \neq b$ implies $f(a) \neq f(b)$ and $a<b$ implies $f(a)<f(b)$. The universal set is a generalization of the Cantor discontinuum to the transfinite. (Received April 10, 1939.)

273. Oystein Ore and J. E. Eaton: *Remarks on multigroups.*

This paper contains various contributions which lead to simplifications and improvements in certain parts of the previous theory of multigroups. (Received April 25, 1939.)

274. J. F. Ritt: *On intersections of algebraic differential manifolds.*

An example is presented which exhibits a fundamental difference between the dimensionality theory of algebraic manifolds and that of differential manifolds. In this example, three unknown functions are uniquely determined by what may quite fairly be called two conditions. (Received April 5, 1939.)

275. J. F. Ritt and E. R. Kolchin: *On certain ideals of differential polynomials.*

This note deals with the decomposition theory of ideals whose manifolds are composed of several disjoint submanifolds. An examination is made of the case in which one of the submanifolds consists of a single solution. (Received April 5, 1939.)

276. L. D. Rodabaugh: *On the partial differential equation* $\partial z/\partial x + f(x,y)\partial z/\partial y = 0$.

Theorem 1. Assume that g is a bounded, open, simply connected plane region and $f(x, y)$ is a function such that (a) $f(x, y)$ and $f_y(x, y)$ are defined and are continuous in g; (b) $f(x, y)$ and $f_y(x, y)$ have definite finite continuous limits on the boundary of g. Assume also that L and U are any finite real numbers such that L is less than U. Then there exists a function $I(x, y)$ such that, in g: (a) $I(x, y)$ is defined and is of class C' with respect to x and y; (b) $I(x, y)$ satisfies the partial differential equation $\partial z/\partial x + f(x, y)\partial z/\partial y = 0$; (c) $I_y(x, y) > 0$; (d) $L < I(x, y) < U$. Theorem 2 generalizes Theorem 1 to the case of a doubly connected region; extension is also made to the case of a multiply connected region of finite order of connectivity greater than two. The paper deals only with real functions of real variables. (Received April 1, 1939.)

277. W. T. Scott and H. S. Wall: *A convergence theorem for continued fractions.*

The continued fraction $[a_n/1]_1^\infty$, in which the a_n are complex numbers, converges if there exists a convergent series of positive numbers $p_1+p_2+p_3+\cdots$ such that $r_1|1+a_2| \geq |a_2|$, $r_2|1+a_2+a_3| \geq |a_3|$; $r_n|1+a_n+a_{n+1}| \geq r_n r_{n-2}|a_n| + |a_{n+1}|$, $(n=3, 4, 5, \cdots)$, where $r_n = p_{n+1}/p_n$, $(n=1, 2, 3, \cdots)$. The power of this theorem is illustrated by the following result which is obtained from it: The continued fraction $[a_n/1]_1^\infty$ converges if the a_n are contained in any closed bounded region lying within the parabola $|z| - R(z) = 1/2$. That this is the best such theorem about regions containing the origin and symmetric with respect to the real axis is easily seen since the continued fraction $a/1+\bar{a}/1+a/1+\bar{a}/1+\cdots$ diverges when a is outside the parabola. (Received May 12, 1939.)

278. M. F. Smiley: *A note on measure functions in a lattice.*

This note presents a generalization of Carathéodory's criterion of measurability which is applicable to an arbitrary lattice. It is shown that for modular lattices the "measurable" elements form a sublattice. The question of closure of this sublattice under denumerable sums and products is discussed. Some of the properties of regular real valued lattice functions (cf. Carathéodory, *Vorlesungen über reelle Funktionen*, 2d edition, p. 258) are derived. (Received May 1, 1939.)

279. A. D. Wallace: *An axiomatic treatment of separation.* Preliminary report.

In a space S, take as undefined the notion of two point sets being mutually separated. To indicate this relation, write $X|Y$. The following conditions are assumed: (1) $X|Y$ implies $Y|X$; (2) $X|Y$ implies $X \neq 0 \neq Y$; (3) $0 \neq X \subset Y$ and $Y|Z$ imply $X|Z$; (4) $X|Y$ and $X|Z$ imply $X|(Y+Z)$; (5) $X|Y$ implies $XY=0$. A set is said to be connected provided that it is not the sum of two mutually separated sets.

On the basis of the above axioms, it does not follow that two distinct points are mutually separated. Nevertheless a large body of theorems concerning connected sets holds true. For example, the following result is obtained: If X is a connected subset of the connected set Y and $Y-X=Y_1+Y_2$ where $Y_1 Y_2$, then Y_1+X and Y_2+X are connected; and if Z is a component of $Y-X$, then $Y-Z$ is connected. There are various ways in which a topology may be introduced into S. For a "closure" topology, define the closure of the set X to be the set of all points not separated from X. It is also possible to introduce a "neighborhood" topology in several different ways. The space S may be considered as a "discrete space" (Linfield) or as a space of "contiguous points" (R. L. Moore). (Received April 6, 1939.)

280. Morgan Ward: *An arithmetical characterization of a modular lattice.*

Let $\mathfrak{S}$ be a lattice with respect to a division relation $x \supset y$, and let $[x, y]$ denote the crosscut of x and y. An element q of $\mathfrak{S}$ is said to be irreducible if $q=[v, w]$ implies $q=v$ or $q=w$. An irreducible is said to be a component of an element u of $\mathfrak{S}$ if $u=[q, v]$ and $q \not\supset v$. If q is a component of u, any element $\bar{q}$ such that $u=[q, \bar{q}]$, $q \not\supset \bar{q}$ is called a complement of q in u. It is proved in this paper that if the ascending chain condition holds in $\mathfrak{S}$, then the following condition is both necessary and sufficient that $\mathfrak{S}$ be a modular lattice: If q is irreducible, and c any element such that neither $q \supset c$ nor $c \supset q$, then q is a component of every element d of the quotient lattice $q/[q, c]$; and if $d \neq q$, q has at least one complement in d dividing c. (Received April 3, 1939.)

281. G. T. Whyburn: *Non-alternating interior retracting transformations.*

Let M be a compact locally connected continuum, let axb be any simple arc in M, and let J be any simple closed curve in M. It is shown that there exists a non-alternating transformation which retracts M into axb and is interior on the cyclic chain $C(a, b)$. Similarly, if M is not unicoherent about J, there exists a non-alternating transformation which retracts M into J and is interior on the cyclic element $C(J)$ of M containing J. Also it is shown that M will be mappable onto a circle by a non-alternating interior transformation if and only if M is cyclic and non-unicoherent. (Received April 12, 1939.)

282. L. E. Mehlenbacher: *Determination of the asymptotic behavior of the solutions of differential equations of the Fuchsian type; the case of $n+2$ regular singular points.*

This paper deals with the linear homogeneous differential equation of the second order $z^2[a_\alpha z^\alpha]d^2y/dz^2+z[b_\alpha z^\alpha]dy/dz+c_\alpha z^\alpha y=0$, where α denotes summation from 1 to n. According to the Fuchsian theory, this equation has $n+2$ regular singular points, $z=0$, $z=\infty$, and the n zeros $z=z_i$, $(i=1, 2, 3, \cdots, n)$, of $a_\alpha z^\alpha=0$, at each of which there exist two solutions each expressible in an infinite series. The solutions at $z=0$ are denoted by y_1, y_2 and those about $z=\infty$ by $\bar{y}_1$, $\bar{y}_2$. The problem of the paper is to determine the precise manner in which each of the solutions y_1, y_2 is connected linearly with the two solutions $\bar{y}_1$, $\bar{y}_2$; similar relations for the solutions about the points $z=z_i$ being obtained by similar methods. Methods used are based upon the work of Nörlund in difference equations. The results generalize those of W. B. Ford, who treated the case in which $n=2$. (Received May 13, 1939.)

283. Barkley Rosser: *A new lower bound for the exponent in the first case of Fermat's last theorem.*

This paper proves the theorem: If p is an odd prime and $a^p+b^p+c^p=0$ has a solution in integers prime to p, then $p>41{,}000{,}000$. It seems certain that still higher lower bounds for p can be deduced by the methods of this paper. However, an argument is given which makes it seem unlikely that an indefinitely high lower bound can be so deduced. (Received May 15, 1939.)

284. O. F. G. Schilling: *Regular normal extensions over complete fields.*

Let k be a complete discrete field whose residue class field is a finite field of characteristic χ. Suppose that $\chi \equiv 1 \pmod{p}$, p a prime. The author investigates regular normal extensions $K|k$, that is, extensions whose Galois groups have orders p^n. It is shown that the Galois group of the join $\sum K$ contains a well-defined everywhere dense subgroup F. This group F is a discrete group which suffices to describe the Galois theory of $K|k$. It turns out that F may be considered as the generalization of the Fuchsian groups occurring in the theory of algebraic functions. Finally, the Galois groups of certain regular extensions are interpreted in terms of factor groups taken from normal division algebras over k. (Received May 16, 1939.)

285. W. T. Scott and H. S. Wall: *A convergence theorem for continued fractions.* II.

Employing the theorem, which we recently announced, that the continued fraction (1) $1/1+a_2/1+a_3/1+\cdots$ (the a_n arbitrary complex numbers) converges if there exists a positive term convergent series $p_1+p_2+p_3+\cdots$ such that (2) $r_n|1+a_n+a_{n+1}| \geqq r_n r_{n-2}|a_n|+|a_{n+1}|$, $(n=1, 2, 3, \cdots;\ r_{-1}=r_0=a_1=0)$, where $r_n=p_{n+1}/p_n$, the authors find the following results: (a) Let $P(x, y)$ be any point on the curve $y=\pm(2x+1)(4x+1)^{1/2}/2x$, O the origin, and Q the point $z=x$, in the complex plane of $z=x+iy$. Then (1) converges if the a_n lie in any closed region interior to the triangle OPQ. (b) If (2) holds for real positive numbers r_n (not necessarily related to a convergent series $\sum p_n$), then the sequences of even and odd convergents of (1) have limits, finite or infinite. The limit of the sequence of even (odd) convergents is finite if actual inequality holds in (2) for an odd (even) value of n. (Received May 18, 1939.)

286. W. E. Sewell: *Continuity and degree of approximation by rational functions.*

This paper deals with the relation between the degree of approximation by rational functions with exterior poles and the continuity properties of the function approximated. (Received May 15, 1939.)

287. W. E. Sewell: *Continuity and integral approximation to an analytic function by polynomials in z and $1/z$.*

Let C be a Jordan curve in the z-plane, and let $f(z)$ be defined on C. This paper deals with the relation between the continuity properties of $f(z)$ on C and the degree of approximation to $f(z)$ by polynomials in z and $1/z$ as measured by a line integral over C. (Received May 15, 1939.)

288. W. E. Sewell: *The derivative of a polynomial on further arcs of the complex domain.*

In this paper results already established by the author (American Mathematical Monthly, vol. 44 (1937), pp. 577–578; National Mathematics Magazine, vol. 12 (1938), pp. 167–170) for certain curves and arcs of the complex domain are extended to the limaçon and the lemniscate. (Received May 15, 1939.)

289. Morgan Ward: *The ideal operators of a lattice.*

Let $\mathfrak{S}$ be a closed lattice. An operator $\theta S = \theta(S)$ on $\mathfrak{S}$ to $\mathfrak{S}$ is called an "ideal operator" if (i) $A \supset B$ implies $\theta A \supset \theta B$; (ii) $\theta A \supset A$; (iii) $\theta(\theta A) = \theta A$. The set $\mathfrak{S}'$ of values θS of θ is closed under crosscut and contains the unit I of $\mathfrak{S}$. It may be made into a closed lattice within $\mathfrak{S}$ by assigning as union, to any set of values V, the crosscut of all values containing every V. Conversely to any set $\mathfrak{S}'$ closed under crosscut and containing I there corresponds an ideal operator θ with values $\mathfrak{S}' = \theta\mathfrak{S}$. If O is any semi-ordered set, we introduce ideal operators θ in the Boolean algebra $\mathfrak{B}$ of all subsets S of O. To each element a of O corresponds the principal ideal A of all elements t of O such that $a \geqq t$ in O. Every ideal operator θ of $\mathfrak{B}$ leaving all principal ideals of O invariant gives a lattice $\mathfrak{S}' = \theta\mathfrak{B}$ in which O is isomorphically imbedded. We obtain the v-ideals or ovoid ideals of a semi-group as a special case. (Received May 5, 1939.)

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